Mathematical Association of America
American Mathematics Competitions

22nd Annual

AMERICAN INVITATIONAL MATHEMATICS EXAMINATION (AIME)

SOLUTIONS PAMPHLET

Tuesday, March 23, 2004

This Solutions Pamphlet gives at least one solution for each problem on this year's AIME and shows that all the problems can be solved using precalculus mathematics. When more than one solution for a problem is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic vs geometric, computational vs. conceptual, elementary vs. advanced. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers inform their students about these solutions, both as illustrations of the kinds of ingenuity needed to solve nonroutine problems and as examples of good mathematical exposition. Routine calculations and obvious reasons for proceeding in a certain way are often omitted. This gives greater emphasis to the essential ideas behind each solution. *Remember that reproduction of these solutions is prohibited by copyright.*

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1. (Answer: 217)

   Let the digits of n, read from left to right, be a, $a-1$, $a-2$, and $a-3$, respectively, where a is an integer between 3 and 9, inclusive. Then $n = 1000a + 100(a-1) + 10(a-2) + a - 3 = 1111a - 123 = 37(30a-4) + (a+25)$, where $0 \le a + 25 < 37$. Thus the requested sum is

$$\sum_{a=3}^{9}(a+25) = \left(\sum_{a=3}^{9} a\right) + 175 = 42 + 175 = 217.$$

OR

There are seven such four-digit integers, the smallest of which is 3210, whose remainder when divided by 37 is 28. The seven integers form an arithmetic sequence with common difference 1111, whose remainder when divided by 37 is 1, so the sum of the remainders is $28+29+30+31+32+33+34 = 7 \cdot 31 = 217$.

2. (Answer: 201)

   Let the smallest elements of $\mathcal{A}$ and $\mathcal{B}$ be $(n+1)$ and $(k+1)$, respectively. Then

$$2m = (n+1) + (n+2) + \cdots + (n+m) = mn + \frac{1}{2} \cdot m(m+1), \quad \text{and}$$
$$m = (k+1) + (k+2) + \cdots + (k+2m) = 2km + \frac{1}{2} \cdot 2m(2m+1).$$

The second equation implies that $k + m = 0$. Substitute this into $|k + 2m - (n+m)| = 99$ to obtain $n = \pm 99$. Now simplify the first equation to obtain $2 = n + (m+1)/2$, and substitute $n = \pm 99$. This yields $m = -195$ or $m = 201$. Because $m > 0$, $m = 201$.

OR

The mean of the elements in $\mathcal{A}$ is 2, and the mean of the elements in $\mathcal{B}$ is $1/2$. Because the mean of each of these sets equals its median, and the median of $\mathcal{A}$ is an integer, m is odd. Thus $\mathcal{A} = \{2 - \frac{m-1}{2}, \ldots, 2, \ldots, 2 + \frac{m-1}{2}\}$, and $\mathcal{B} = \{-m+1, \ldots, 0, 1, \ldots, m\}$. Therefore $|2 + \frac{m-1}{2} - m| = 99$, which yields $|\frac{3-m}{2}| = 99$, so $|3 - m| = 198$. Because $m > 0$, $m = 201$.

3. (Answer: 241)

The total number of diagonals and edges is $\binom{26}{2} = 325$, and there are $12 \cdot 2 = 24$ face diagonals, so P has $325 - 60 - 24 = 241$ space diagonals. One such polyhedron can be obtained by gluing two dodecahedral pyramids onto the 12-sided faces of a dodecahedral prism.

Note that one can determine that there are 60 edges as follows. The 24 triangles contribute $3 \cdot 24 = 72$ edges, and the 12 quadrilaterals contribute $4 \cdot 12 = 48$ edges. Because each edge is in two faces, there are $\frac{1}{2}(72 + 48) = 60$ edges.

4. (Answer: 086)

Let $\overline{PQ}$ be a line segment in set $\mathcal{S}$ that is not a side of the square, and let M be the midpoint of $\overline{PQ}$. Let A be the vertex of the square that is on both the side that contains P and the side that contains Q. Because $\overline{AM}$ is the median to the hypotenuse of right $\triangle PAQ$, $AM = (1/2) \cdot PQ = (1/2) \cdot 2 = 1$. Thus every midpoint is 1 unit from a vertex of the square, and the set of all the midpoints forms four quarter-circles of radius 1 and with centers at the vertices of the square. The area of the region bounded by the four arcs is $4 - 4 \cdot (\pi/4) = 4 - \pi$, so $100k = 100(4 - 3.14) = 86$.

OR

Place a coordinate system so that the vertices of the square are at $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$. When the segment's vertices are on the sides that contain $(0, 0)$, its endpoints' coordinates can be represented as $(a, 0)$ and $(0, b)$. Let the coordinates of the midpoint of the segment be (x, y). Then $(x, y) = (a/2, b/2)$ and $a^2 + b^2 = 4$. Thus $x^2 + y^2 = (a/2)^2 + (b/2)^2 = 1$, and the midpoints of these segments form a quarter-circle with radius 1 centered at the origin. The set of all the midpoints forms four quarter-circles, and the area of the region bounded by the four arcs is $4 - 4 \cdot (\pi/4) = 4 - \pi$, so $100k = 100(4 - 3.14) = 86$.

5. (Answer: 849)

Let Beta's scores be a out of b on day one and c out of d on day two, so that $0 < a/b < 8/15$, $0 < c/d < 7/10$, and $b + d = 500$. Then $(15/8)a < b$ and $(10/7)c < d$, so $(15/8)a + (10/7)c < b + d = 500$, and $21a + 16c < 5600$. Beta's two-day success ratio is greatest when $a + c$ is greatest. Let $M = a + c$ and subtract $16M$ from both sides of the last inequality to obtain $5a < 5600 - 16M$. Because $a > 0$, conclude that $5600 - 16M > 0$, and $M < 350$. When $M = 349$, $5a < 16$, so $a \le 3$. If $a = 3$, then $b \ge 6$, but then $d \le 494$ and $c = 346$ so $c/d \ge 346/494 > 7/10$. Notice that when $a = 2$ and $b = 4$, then $a/b < 8/15$ and

$c/d = 347/496 < 7/10$. Thus Beta's maximum possible two-day success ratio is $349/500$, so $m + n = 849$.

OR

Let M be the total number of points scored by Beta in the two days. Notice first that $M < 350$, because 350 is 70% of 500, and Beta's success ratio is less than 70% on each day of the competition. Notice next that $M = 349$ is possible, because Beta could score 1 point out of 2 attempted on the first day, and 348 out of 498 attempted on the second day. Thus $m = 349$, $n = 500$, and $m + n = 849$.

Note that Beta's two-day success ratio can be greater than Alpha's while Beta's success ratio is less on each day. This is an example of Simpson's Paradox.

6. (Answer: 882)

To find the number of snakelike numbers that have four different digits, distinguish two cases, depending on whether or not 0 is among the chosen digits. For the case where 0 is not among the chosen digits, first consider only the digits 1, 2, 3, and 4. There are exactly 5 snakelike numbers with these digits: 1423, 1324, 2314, 2413, and 3412. There are $\binom{9}{4} = 126$ ways to choose four non-zero digits and five ways to arrange each such set for a total of 630 numbers. In the other case, there are $\binom{9}{3} = 84$ ways to choose three digits to go with 0, and three ways to arrange each set of four digits, because the snakelike numbers with the digits 0, 1, 2, and 3 would correspond to the list above, but with the first two entries deleted. There are $84 \cdot 3 = 252$ such numbers. Thus there are $630 + 252 = 882$ four-digit snakelike numbers with distinct digits.

7. (Answer: 588)

Each of the x^2-terms in the expansion of the product is obtained by multiplying the x-terms from two of the 15 factors of the product. The coefficient of the x^2-term is therefore the sum of the products of each pair of numbers in the set $\{-1, 2, -3, \ldots, 14, -15\}$. Note that, in general,

$$(a_1 + a_2 + \cdots + a_n)^2 = a_1^2 + a_2^2 + \cdots + a_n^2 + 2 \cdot \left(\sum_{1 \le i < j \le n} a_i a_j \right).$$

Thus

$$C = \sum_{1 \le i < j \le 15} (-1)^i i (-1)^j j = \frac{1}{2}\left(\left(\sum_{k=1}^{15}(-1)^k k\right)^2 - \sum_{k=1}^{15} k^2\right)$$
$$= \frac{1}{2}\left((-8)^2 - \frac{15(15+1)(2\cdot 15+1)}{6}\right) = -588.$$

Hence $|C| = 588$.

OR

Note that

$$\begin{aligned} f(x) &= (1-x)(1+2x)(1-3x)\ldots(1-15x) \\ &= 1 + (-1+2-3+\cdots-15)x + Cx^2 + \cdots \\ &= 1 - 8x + Cx^2 + \cdots . \end{aligned}$$

Thus $f(-x) = 1 + 8x + Cx^2 - \cdots$.
But $f(-x) = (1+x)(1-2x)(1+3x)\ldots(1+15x)$, so

$$\begin{aligned} f(x)f(-x) &= (1-x^2)(1-4x^2)(1-9x^2)\ldots(1-225x^2) \\ &= 1 - (1^2+2^2+3^2+\cdots+15^2)x^2 + \cdots . \end{aligned}$$

Also $f(x)f(-x) = (1-8x+Cx^2+\cdots)(1+8x+Cx^2-\cdots) = 1+(2C-64)x^2+\cdots$.
Thus $2C - 64 = -(1^2 + 2^2 + 3^3 + \cdots + 15^2)$, and, as above, $|C| = 588$.

8. (Answer: 199)

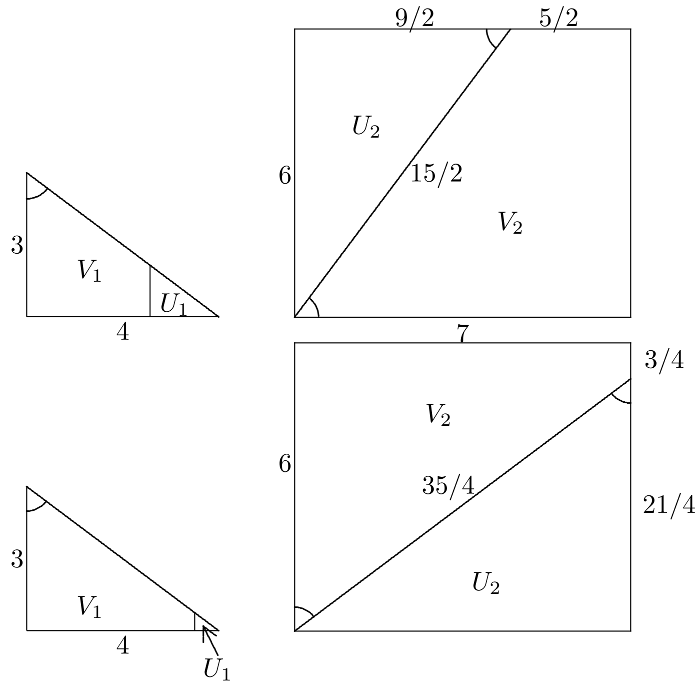
Let $\mathcal{C}$ be the circle determined by P_1, P_2, and P_3. Because the path turns counterclockwise at an angle of less than 180° at P_2 and P_3, P_1 and P_4 must be on the same side of line P_2P_3. Note that $\triangle P_1P_2P_3 \cong \triangle P_4P_3P_2$, and so $\angle P_2P_1P_3 \cong \angle P_3P_4P_2$. Thus P_4 is on $\mathcal{C}$. Similarly, because P_2, P_3, and P_4 are on $\mathcal{C}$, P_5 must be too, and, in general, $P_1, P_2, \ldots, P_n$ are all on $\mathcal{C}$. The fact that the minor arcs $P_1P_2, P_2P_3, \ldots,$ and P_nP_1 are congruent implies that $P_1, P_2, \ldots,$ and P_n are equally spaced on $\mathcal{C}$.

Thus any regular n-pointed star can be constructed by choosing n equally spaced points on a circle, and numbering them consecutively from 0 to $n-1$. For positive integers $d < n$, the path consisting of line segments whose vertices are numbered $0, d, 2d, \ldots, (n-1)d, 0$ modulo n will be a regular n-pointed star if and only if $2 \le d \le n-2$ and d is relatively prime to n. This is because if $d = 1$ or $d = n-1$, the resulting path will be a polygon; and if d is not relatively prime to n, not every vertex will be included in the path. Also, for any choice of d that yields a regular n-pointed star, any two such stars will be similar because a dilation of one of the stars about the center of its circle will yield the other.

Because $1000 = 2^3 \cdot 5^3$, numbers that are relatively prime to 1000 are those that are multiples of neither 2 nor 5. There are $1000/2 = 500$ multiples of 2 that are less than or equal to 1000; there are $1000/5 = 200$ multiples of 5 that are less than or equal to 1000; and there are $1000/10 = 100$ numbers less than or equal to 1000 that are multiples of both 2 and 5. Hence there are $1000 - (500 + 200 - 100) = 400$ numbers that are less than 1000 and relatively prime to 1000, and 398 of them are between 2 and 998, inclusive. Because $d = k$ yields the same path as $d = n - k$ (and also because one of these two paths turns clockwise at each vertex), there are $398/2 = 199$ non-similar regular 1000-pointed stars.

9. (Answer: 035)

Let s_1 be the line segment drawn in $\triangle ABC$, and let s_2 be the line segment drawn in rectangle $DEFG$. To obtain a triangle and a trapezoid, line segment s_2 must pass through exactly one vertex of rectangle $DEFG$. Hence V_2 is a trapezoid with a right angle, and U_2 is a right triangle. Therefore line segment s_1 is parallel to one of the legs of $\triangle ABC$ and, for all placements of s_1, U_1 is similar to $\triangle ABC$. It follows that there are two possibilities for triangle U_2: one in which the sides are 6, 9/2, and 15/2; and the other in which the sides are 7, 21/4, and 35/4. Were s_1 parallel to the side of length 4, trapezoids V_1 and V_2 could not be similar, because the corresponding acute angles in V_1 and V_2 would not be congruent; but when s_1 is parallel to the side of length 3, the angles of trapezoid V_1 are congruent to the corresponding angles of V_2, so it is possible to place segment s_1 so that V_1 is similar to V_2. In the case when the triangle U_2 has sides 6, 9/2, and 15/2, the bases of trapezoid V_2 are 7 and $7 - (9/2) = 5/2$, so its bases, and therefore the bases of V_1, are in the ratio $5:14$. Then the area of triangle U_1 is $(5/14)^2 \cdot (1/2) \cdot 3 \cdot 4 = 75/98$. In the case when the triangle U_2 has sides 7, 21/4, and 35/4, the bases of trapezoid V_2 are 6 and $6 - (21/4) = 3/4$, so its bases, and the bases of V_1, are in the ratio $1:8$. The area of triangle U_1 is then $(1/8)^2 \cdot (1/2) \cdot 3 \cdot 4 = 3/32$. The minimum value of the area of U_1 is thus $3/32$, and $m + n = 35$.



10. (Answer: 817)

In order for the circle to lie completely within the rectangle, the center of the circle must lie in a rectangle that is $(15-2)$ by $(36-2)$ or 13 by 34. The requested probability is equal to the probability that the distance from the circle's center to the diagonal $\overline{AC}$ is greater than 1, which equals the probability that the distance from a randomly selected point in the 13-by-34 rectangle to each of the sides of $\triangle ABC$ and $\triangle CDA$ is greater than 1. Let $AB = 36$ and $BC = 15$. Draw the three line segments that are one unit respectively from each of the sides of $\triangle ABC$ and whose endpoints are on the sides. Let E, F, and G be the three points of intersection nearest to A, B, and C, respectively, of the three line segments. Let P be the intersection of $\overrightarrow{EG}$ and $\overline{BC}$, and let G' and P' be the projections of G and P on $\overline{BC}$ and $\overline{AC}$, respectively. Then $FG = BC - CP - PG' - 1$. Notice that $\triangle PP'C \sim \triangle ABC$ and $PP' = 1$, so $CP = AC/AB$. Similarly, $\triangle GG'P \sim \triangle ABC$ and $GG' = 1$, so $PG' = CB/AB$.

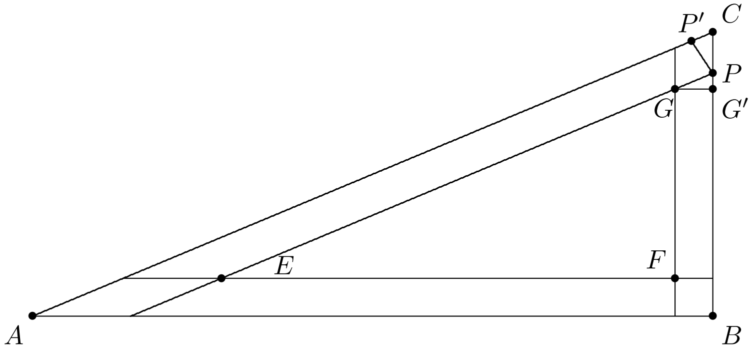
Thus

$$FG = BC - \frac{AC}{AB} - \frac{CB}{AB} - 1.$$

Apply the Pythagorean Theorem to $\triangle ABC$ to obtain $AC = 39$. Substitute these lengths to find that $FG = 25/2$. Notice that $\triangle EFG \sim \triangle ABC$, and their similarity ratio is $(25/2)/15 = 5/6$, so $[EFG] = (25/36)[ABC]$. The requested probability is therefore

$$\frac{2 \cdot \frac{25}{36} \cdot \frac{1}{2} \cdot 15 \cdot 36}{13 \cdot 34} = \frac{375}{442},$$

so $m + n = 817$.



OR

Define E, F, and G as in the previous solution. Each of these three points is equidistant from two sides of $\triangle ABC$, and they are therefore on the angle-bisectors of angles A, B, and C, respectively. These angle-bisectors are also angle-bisectors of $\triangle EFG$ because its sides are parallel to those of $\triangle ABC$. Thus $\triangle ABC$ and $\triangle EFG$ have the same incenter, and the inradius of $\triangle EFG$ is one less than that of $\triangle ABC$. In general, the inradius of a triangle is the area divided by one-half the perimeter, so the inradius of $\triangle ABC$ is 6. The similarity ratio of $\triangle EFG$ to $\triangle ABC$ is the same as the ratio of their inradii, namely $5/6$. Continue as in the previous solution.

OR

Define E, F, G, and G' as in the previous solutions. Notice that $\overline{CG}$ bisects $\angle ACB$ and that $\cos \angle ACB = 5/13$, and so, by the Half-Angle Formula, $\cos \angle GCG' = 3/\sqrt{13}$. Thus, for some x, $CG' = 3x$ and $CG = x\sqrt{13}$. Apply the Pythagorean Theorem in $\triangle CG'G$ to conclude that $(x\sqrt{13})^2 - (3x)^2 = 1$, so $x = 1/2$. Then $CG' = 3x = 3/2$, and $FG = 15 - 1 - 3/2 = 25/2$. Continue as in the first solution.

11. (Answer: 512)

The lateral surface area of a cone with radius R and slant height S can be found by cutting the cone along a slant height and then unrolling it to form a sector of a circle. The sector's arc has length $2\pi R$ and its radius is S, so its area, and the cone's lateral surface area, is $\pi S^2 \cdot \frac{2\pi R}{2\pi S} = \pi RS$. Let r, h, and s represent the radius, height, and slant height of the smaller cone formed by the cut. Then

$$\begin{aligned} k &= \frac{r^2 h}{36 - r^2 h} = \frac{rs}{9 + 15 - rs}, \quad \text{so} \\ \frac{1}{k} &= \frac{36}{r^2 h} - 1 = \frac{24}{rs} - 1. \end{aligned}$$

Thus $3s = 2rh$. Because $r : h : s = 3 : 4 : 5$, let $(r, h, s) = (3x, 4x, 5x)$, and substitute to find that $x = 5/8$, and then that $(r, h, s) = (15/8, 20/8, 25/8)$. The ratio of the volume of $\mathcal{C}$ to that of the large cone is therefore $\left(\frac{15/8}{3}\right)^3 = \frac{125}{512}$, so the ratio of the volumes of $\mathcal{C}$ and $\mathcal{F}$ is $125/(512 - 125) = 125/387$. Thus $m + n = 125 + 512 - 125 = 512$.

12. (Answer: 014)

Because $\lfloor \log_2(\frac{1}{x}) \rfloor = 2k$ for nonnegative integers k, conclude that $2k \le \log_2(\frac{1}{x}) < 2k + 1$, so

$$2^{2k} \le \frac{1}{x} < 2^{2k+1}, \quad \text{and} \quad \frac{1}{2^{2k+1}} < x \le \frac{1}{2^{2k}}.$$

Similarly, for nonnegative integers k,

$$\frac{1}{5^{2k+1}} < y \le \frac{1}{5^{2k}}.$$

The graph consists of the intersection of two sets of rectangles. The rectangles in one set have vertical sides of length 1 and horizontal sides of lengths $(1 - \frac{1}{2}), (\frac{1}{4} - \frac{1}{8}), (\frac{1}{16} - \frac{1}{32}), \ldots$, and the rectangles in the other set have horizontal sides of length 1 and vertical sides of lengths $(1 - \frac{1}{5}), (\frac{1}{25} - \frac{1}{125}), (\frac{1}{625} - \frac{1}{3125}), \ldots$. The intersection of the two sets of rectangles is also a set of rectangles whose total area is

$$\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{4} - \frac{1}{8}\right) + \cdots\right] \cdot \left[\left(1 - \frac{1}{5}\right) + \left(\frac{1}{25} - \frac{1}{125}\right) + \cdots\right] = \frac{2}{3} \cdot \frac{5}{6} = \frac{5}{9},$$

so $m + n = 14$.

13. (Answer: 482)

Note that for $x \neq 1$,

$$\begin{aligned} P(x) &= \left(\frac{x^{18}-1}{x-1}\right)^2 - x^{17} \quad \text{so} \\ (x-1)^2P(x) &= (x^{18}-1)^2 - x^{17}(x-1)^2 \\ &= x^{36} - 2x^{18} + 1 - x^{19} + 2x^{18} - x^{17} \\ &= x^{36} - x^{19} - x^{17} + 1 \\ &= x^{19}(x^{17}-1) - (x^{17}-1) \\ &= (x^{19}-1)(x^{17}-1), \quad \text{and so} \\ P(x) &= \frac{(x^{19}-1)(x^{17}-1)}{(x-1)^2}. \end{aligned}$$

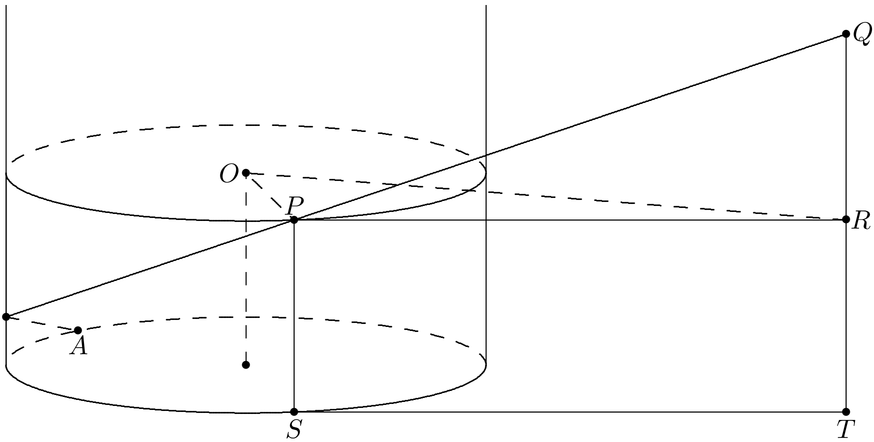
Thus the zeros of $P(x)$ are the 34 complex numbers other than 1 which satisfy $x^{17} = 1$ or $x^{19} = 1$. It follows that $\alpha_1 = 1/19$, $\alpha_2 = 1/17$, $\alpha_3 = 2/19$, $\alpha_4 = 2/17$, and $\alpha_5 = 3/19$, so $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 159/323$, and $m + n = 482$.

14. (Answer: 813)

Suppose that the rope is attached to the ground at point A, last touches the tower at point P, and attaches to the unicorn at point Q. Let S and T be on the ground directly below P and Q, respectively. Let O be on the axis of the tower, and let R be directly below Q so that the plane of $\triangle OPR$ is horizontal. Then $\overline{OP}$ is a radius of the tower, so $OP = 8$, and, because Q is 4 feet from the tower, R is too, so $OR = 12$. Also, $\angle OPR$ is a right angle, so $PR = 4\sqrt{5}$. If the tower wall were spread flat in the plane of P, Q, S, and T, then right triangles PQR and AQT would be similar. Because

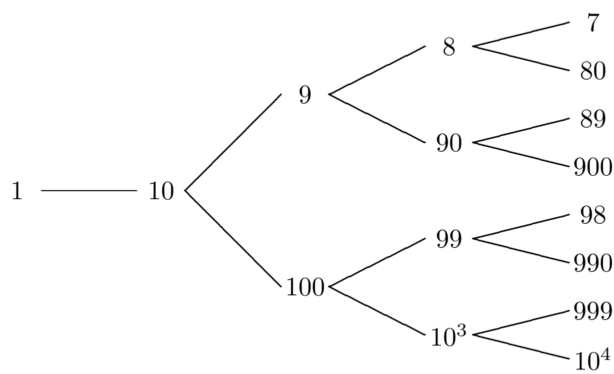
$$\frac{PQ}{PR} = \frac{AQ}{AT} = \frac{20}{\sqrt{20^2 - 4^2}} = \frac{5}{\sqrt{5^2 - 1^2}} = \frac{5}{2\sqrt{6}},$$

where AQ is the rope's length between A and Q and AT is the length of the projection of the rope onto the ground, $PQ = \dfrac{5}{2\sqrt{6}} \cdot 4\sqrt{5} = \dfrac{5\sqrt{30}}{3}$. Then the length of rope touching the tower is $20 - \dfrac{5\sqrt{30}}{3} = \dfrac{60 - \sqrt{750}}{3}$. Thus $a + b + c = 60 + 750 + 3 = 813$.



15. (Answer: 511)

For $i = 1, 2, \ldots$, let $\mathcal{S}_i$ denote the set of positive integers x such that $d(x) = i$. Then, for example, $\mathcal{S}_1 = \{1\}$, $\mathcal{S}_2 = \{10\}$, $\mathcal{S}_3 = \{9, 100\}$, and $\mathcal{S}_4 = \{8, 90, 99, 1000\}$. This can be illustrated in a tree diagram, as shown.



If each vertex in this tree after the first column had two branches, then the 20th column would have 2^{18} vertices; but some vertices have only one branch, namely, the vertex that corresponds to 2 and vertices that correspond to numbers with last digit 1. The vertex that corresponds to 2 is in the 10th column. This causes there to be 2^9 fewer vertices in the 20th column than there would be if each vertex in the tree had two branches. Note that vertices that correspond to numbers with last digit 1 occur 9 columns after vertices that correspond to numbers with last digit 0, except for 10. Thus there will be one 1 in column 12, two 1's in column 13, and in general, 2^{k-12} 1's in column k, for $k = 12, 13, \ldots, 19$. For each of these eight columns, this causes there to be 2^7 fewer vertices in the 20th column than there would be if each vertex in the tree had two branches. Thus m is the number of elements in $\mathcal{S}_{20}$, that is, the number of vertices in column 20, namely,

$$2^{18} - 2^9 - 8 \cdot 2^7 = 2^{18} - 2^9 - 2^{10} = 2^9(2^9 - 1 - 2) = 2^9 \cdot 509,$$

and the sum of the distinct prime factors of m is $2 + 509 = 511$.

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