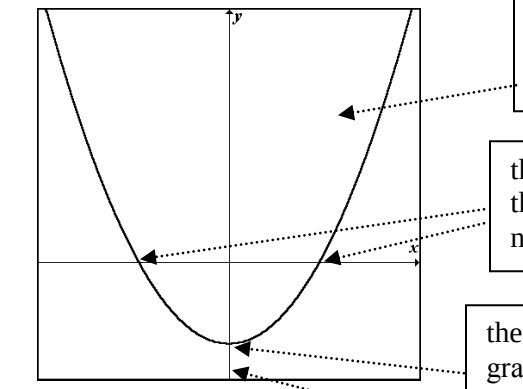
Quadratic Relations

Date:

A quadratic relationship is modeled by $y = ax^2 + bx + c$ when this relationship is graphed; the graph of a quadratic relationship is called a ***parabola.***

The general shape of a quadratic relation is shown in the sketches below:



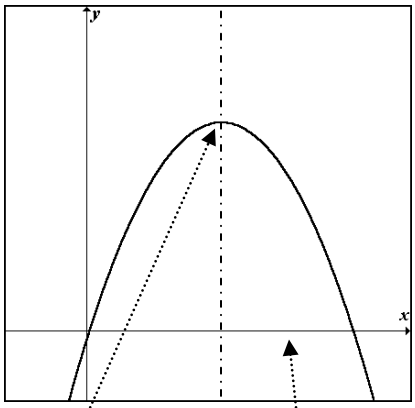
direction of opening – this parabola ***opens upwards.***
- 2nd differences are positive

the ***x-intercepts*** or the ***zeros*** of the relation. These values may not exist.

the ***vertex*** – the point where the graph "changes direction"

axis of symmetry

when a parabola opens upwards, the ***y co-ordinate*** of the vertex represents the ***minimum value*** of the relation. It is the lowest point on the graph.



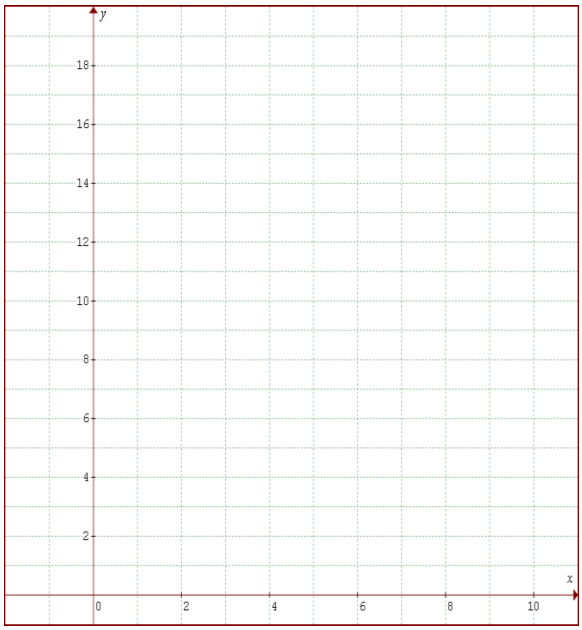
this parabola ***opens downwards***
- 2nd differences negative

the *y* co-ordinate of this vertex would represent the ***maximum value*** of this relation. It is the highest point on this graph.

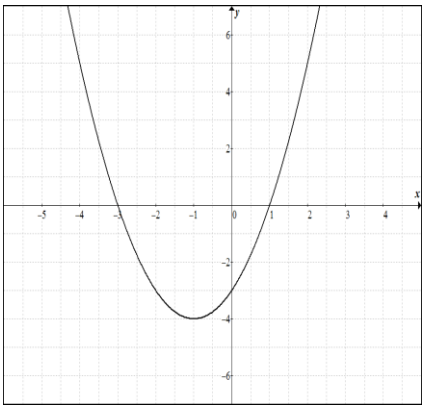
Example 1: Table of values

Graph the data and determine if it is a quadratic relation.

x	y	1st diff	2nd diff
0	16		
1	9		
2	4		
3	1		
4	0		
5	1		
6	4		
7	9		
8	16		



Example 2: Key features of a Quadratic Relation given a graph



Identify the following for the quadratic relation shown:

a) the coordinate of the vertex

b) the equation of the axis of symmetry

c) the *y*-intercept

d) optimum point

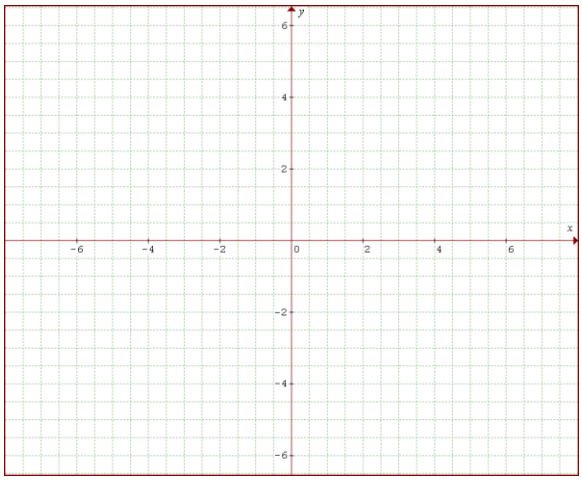
e) the optimum (maximum or minimum) value

f) the *x*-intercept

Example 3: Quadratic Relation in $y=ax^2+b$

Determine the vertex, axis of symmetry, the zeros (*x*-intercepts), and *y*-intercept for the following quadratic relation. Sketch the parabolas.

a) $y = x^2 - 4$



b) $y = -2x^2 + 50$

Exercise

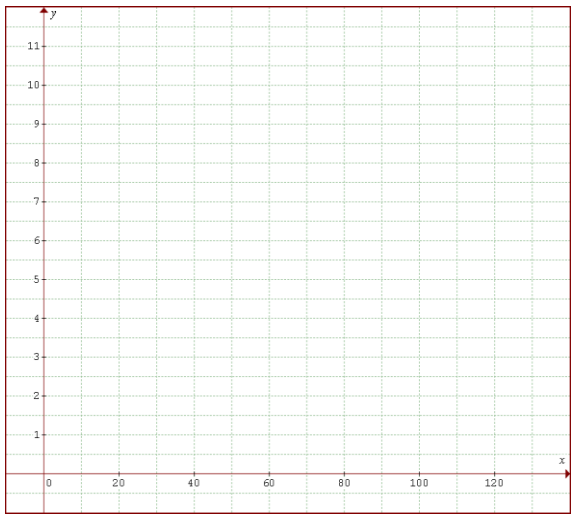
1) For each of the following questions, complete the table by calculating the first and second differences. Use the calculator to create a scatter plot. Sketch the scatter plot in the space provided.

a) the height of a rocket

time	height	first difference	second difference
0	0		
1	25		
2	40		
3	45		
4	40		
5	25		
6	0		

b) the cost of gas

speed car is driven	cost of gas in cents/km	first difference	second difference
20	9.2		
40	7.8		
60	7.1		
80	7.1		
100	7.8		
120	9.2		



2) How can the direction of opening be determined from the table of values *without drawing the graph*?

3) For each of the following, complete the table of values for the given equation. Use the *"y ="* menu on the graphing calculators to draw a graph of each. Copy the graphs onto the graph paper shown. Draw all graphs on the same set of axes.

a)

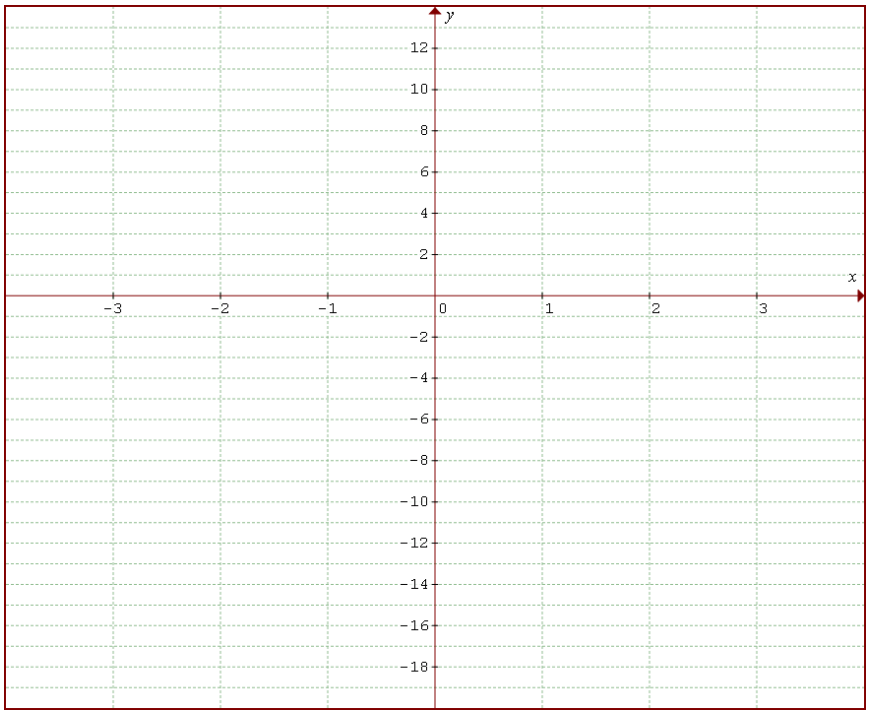
x	$y = x^2$
-3	
-2	
-1	
0	
1	
2	
3	

b)

x	$y = \frac{1}{2}x^2$
-3	
-2	
-1	
0	
1	
2	
3	

c)

x	$y = -2x^2$
-3	
-2	
-1	
0	
1	
2	
3	



4) How can the direction of opening be determined from the equation?

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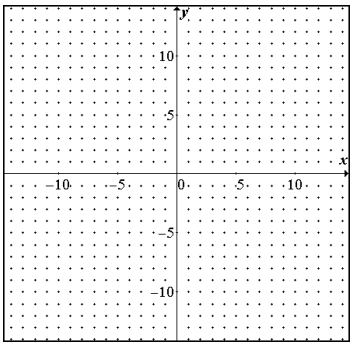
5) For each of the following equations, use the graphing calculator to help you draw a sketch. It is important that you set the "WINDOW" to match the axes shown in the graph paper. Try to determine the co-ordinates of the vertex and the values of the *x*-intercepts and *y*-intercept **from the graphs**.

a) $y = x^2 - 4$



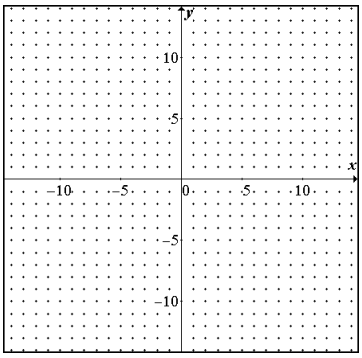
vertex _____

x-intercepts _____

y-intercept _____

axis of symmetry : _____

b) $y = x^2 - 9$



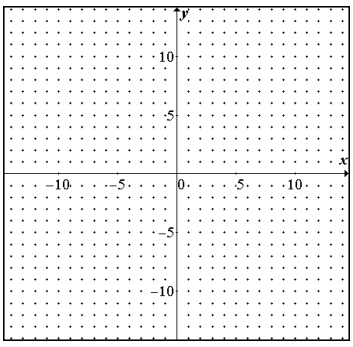
vertex_____

x-intercepts _____

y-intercept _____

axis of symmetry : _____

c) $y = 8 - 2x^2$



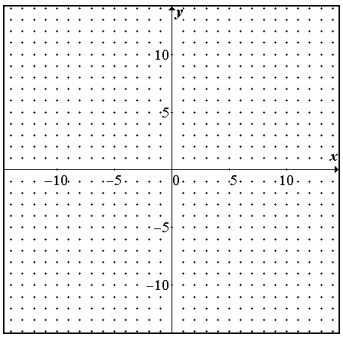
vertex _____

x-intercepts _____

y-intercept _____

axis of symmetry : _____

d) $y = 1 - x^2$



vertex_____

x-intercepts _____

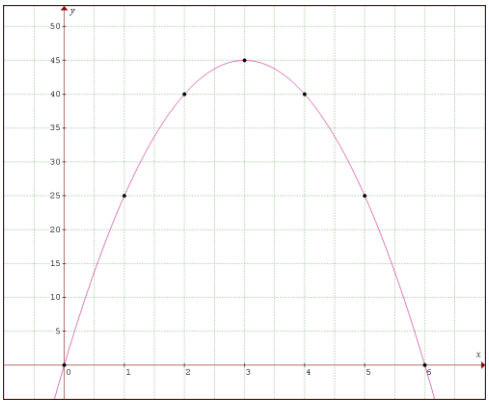
y-intercept _____

axis of symmetry : _____

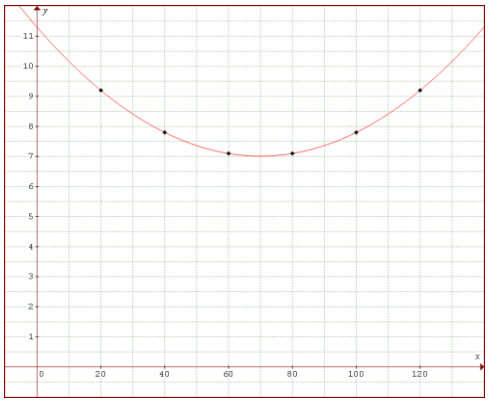
6) What do you notice for the quadratic relation in the form of $y = ax^2 + b$?

Answers

1a)

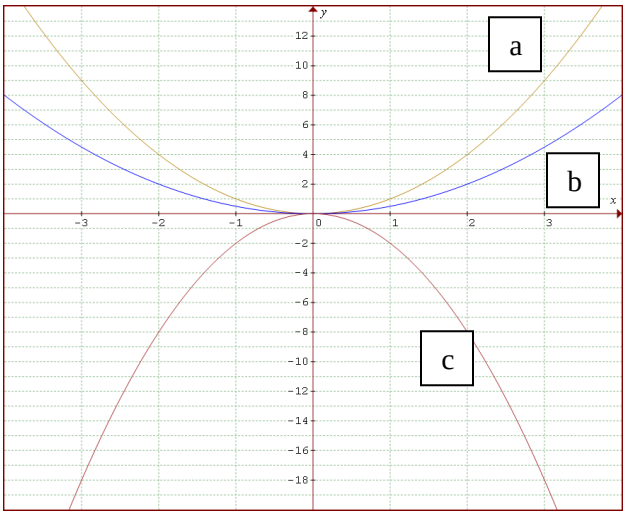


b)



2) Based on the sign of the second differences. If it is positive, opens up, negative, opens down.

3)



4) Based on the sign of the leading coefficient . If it is positive, opens up, negative, opens down.

5a) vertex : $(0,-4)$
$x-\text{int}: (-2,0)\ \&\ (2,0)$
$y-\text{int}: (0,-4)$
Axis of symmetry: $x = 0$

b) vertex : $(0,-9)$
$x-\text{int}: (-3,0)\ \&\ (3,0)$
$y-\text{int}: (0,-9)$
Axis of symmetry: $x = 0$

c) vertex : $(0,8)$
$x-\text{int}: (-2,0)\ \&\ (2,0)$
$y-\text{int}: (0,8)$
Axis of symmetry: $x = 0$

d) vertex : $(0,1)$
$x-\text{int}: (-1,0)\ \&\ (1,0)$
$y-\text{int}: (0,1)$
Axis of symmetry: $x = 0$

6) Vertex: $(0, b)$

x - int: Solve x by letting $y = 0$

or $x = \pm\sqrt{\dfrac{y-b}{a}}$

y – int: $y = b$