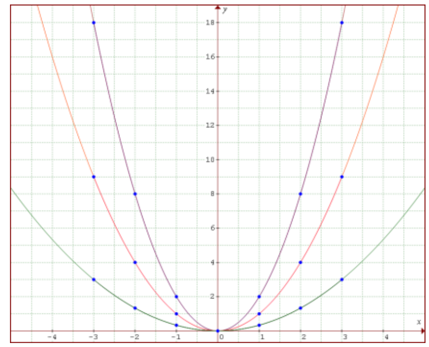
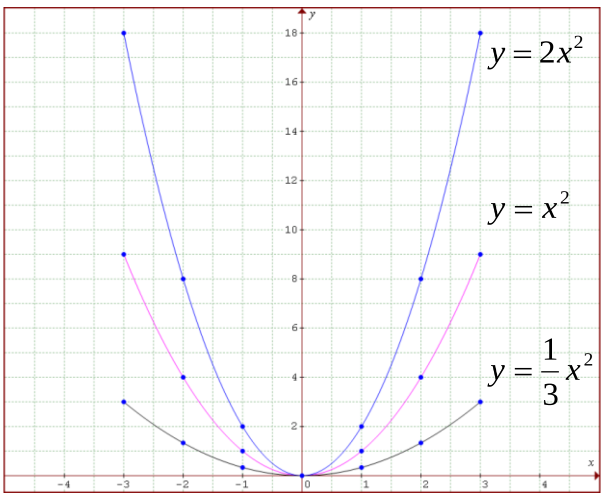
Vertical stretches of a Quadratic Relation (Parabola)

In general, when the graph of a quadratic relation is **elongated** in one direction, the word **stretch (expand)** is used to describe the transformation. If it is shortened in one direction, the word **compression** is used to describe the transformation. In this course, we are going to focus only on one type of stretches – the **vertical stretches** of a quadratic relations.



$y = ax^2$	**Descriptions**	**Examples**
$a > 1$	Vertically expand (stretch) by a factor of *a*	$y = 2x^2$
$0 < a < 1$	Vertically compress by a factor of *a*	$y = \frac{1}{3}x^2$



x	$y = x^2$
-3	***9***
-2	***4***
-1	***1***
0	***0***
1	***1***
2	***4***
3	***9***

x	$y = 2x^2$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = \frac{1}{3}x^2$
-3	
-2	
-1	
0	
1	
2	
3	

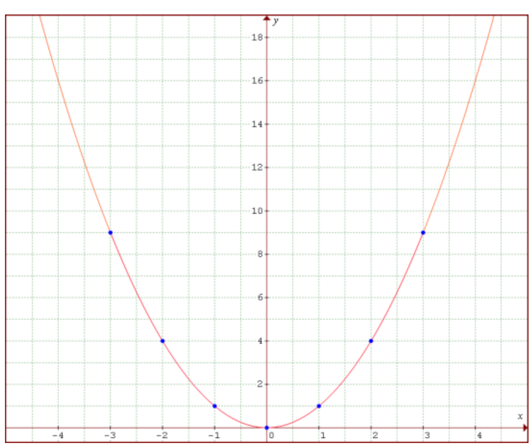
Example 1:

Given the graph of $y = x^2$, complete the graphs of on the same axis. State the mapping rule.

a) $y = 4x^2$ b) $y = (2x)^2$ c) $y = \frac{-1}{2}x^2$

a) $y = 4x^2$

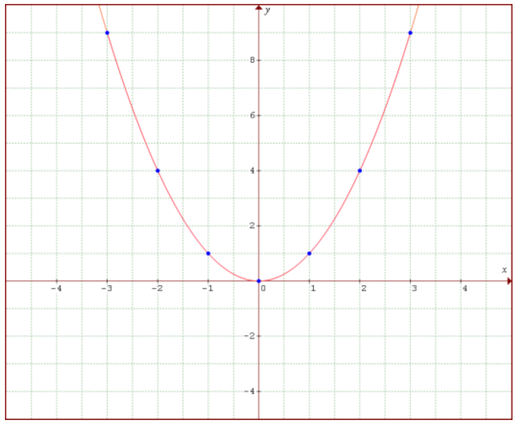
x	
-3	
-2	
-1	
0	
1	
2	
3	



b) $y = (2x)^2$

c) $y = \frac{-1}{2}x^2$

x	$y = \frac{-1}{2}x^2$
-3	
-2	
-1	
0	
1	
2	
3	



Example 2:

Given a parabola in the form of $y = ax^2$, determine the value of "a" if the parabola passes the given points. State the mapping rule.

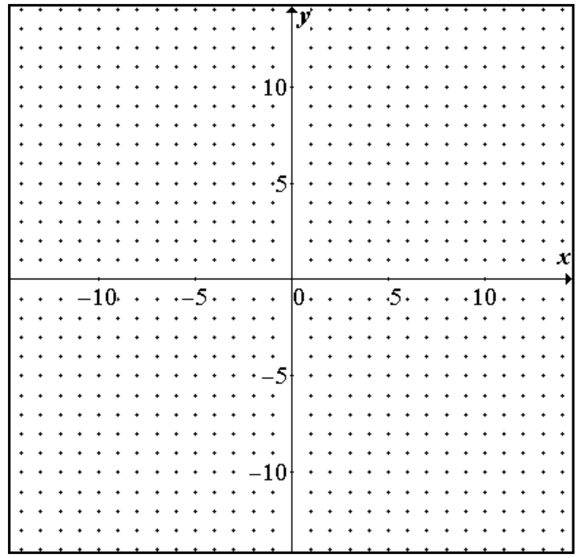
a) $(4, -3)$

b) $\left(\frac{-1}{2}, -5\right)$

Exercise

1. Graph each of the following on the same set of axes. **Label each parabola with its corresponding equation.**

   a) $y = x^2$
   b) $y = 3x^2$
   c) $y = \frac{1}{2}x^2$
   d) $y = -x^2$
   e) $y = -4x^2$
   f) $y = -\frac{1}{3}x^2$



2. Complete the following table:

Equation	co-ordinates of the vertex	direction of opening	x-intercepts	y-intercept
a) $y = x^2$				
b) $y = 3x^2$				
c) $y = \frac{1}{2}x^2$				
d) $y = -x^2$				
e) $y = -4x^2$				
f) $y = -\frac{1}{3}x^2$				

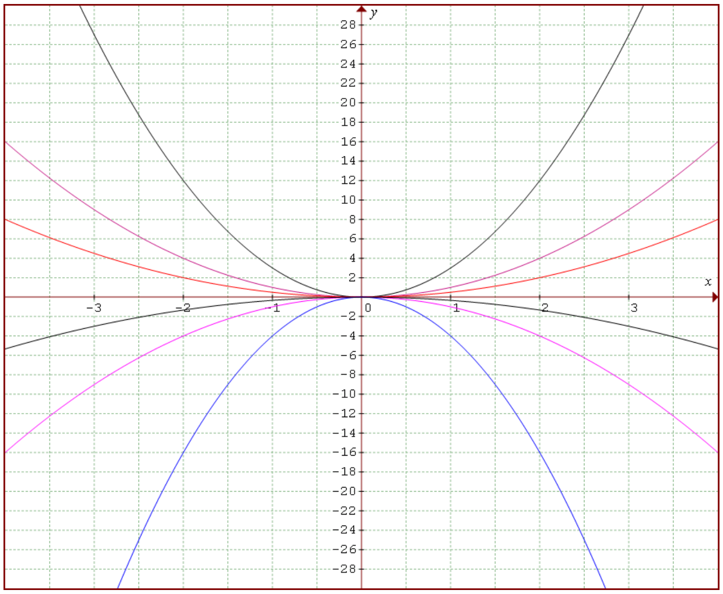
3. Describe the effect on the graph of $y = ax^2$ as the value of "a" varies.

4. All parabolas with the vertex $(0,0)$ have the general equation $y = ax^2$. If we are given a point that the parabola passes through, we can determine the value of 'a'. Determine the equations of the parabolas with vertex (0, 0) that passes through the given points: State the mapping rule for each of them.

   $a)$ (3,18) $b)$ (4,–16) $c)$ (2,24) $d)$ (2,–10) $e)$ $\left(\frac{3}{2},\frac{1}{3}\right)$

Answers

1)



a) $y = x^2$

b) $y = 3x^2$

c) $y = \frac{1}{2}x^2$

d) $y = -x^2$

e) $y = -4x^2$

f) $y = -\frac{1}{3}x^2$

2a) Vertex: (0,0), opens up, x-int: 0, y-int: 0
b) Vertex: (0,0), opens up, x-int: 0, y-int: 0
c) Vertex: (0,0), opens up, x-int: 0, y-int: 0
d) Vertex: (0,0), opens down, x-int: 0, y-int: 0
e) Vertex: (0,0), opens down, x-int: 0, y-int: 0
f) Vertex: (0,0), opens down, x-int: 0, y-int: 0

3) $a > 1$, Vertically expand (stretch) by a factor of a
$0 < a < 1$, Vertically compress by a factor of a

4a) $y = 2x^2$ $\quad (x, y) \to (x, 2y)$

b) $y = -x^2$ $\quad (x, y) \to (x, -y)$

c) $y = y = 6x^2$ $\quad (x, y) \to (x, 6y)$

d) $y = \frac{-5}{2}x^2$ $\quad (x, y) \to \left(x, \frac{-5}{2}y\right)$

e) $y = \frac{4}{27}x^2$ $\quad (x, y) \to \left(x, \frac{4}{27}y\right)$