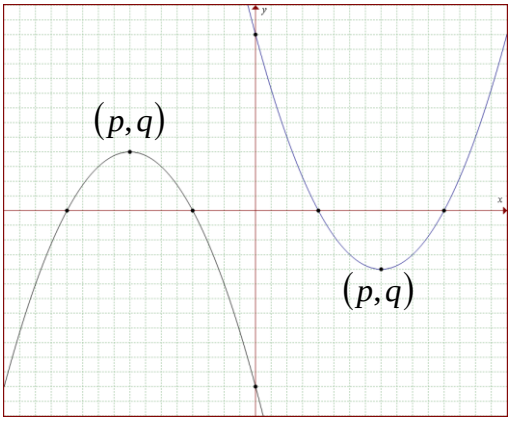
The Quadratic Relation (Vertex Form) – Transformations

Date:

Quadratic Relations in Vertex Form

- The expression $y = a(x-p)^2 + q$ defines a quadratic relation in **vertex form**.
- The coordinates of the vertex of the corresponding parabola are (p,q).
- If $a > 0$, the parabola opens upward. If $a < 0$, the parabola opens downward.
- A quadratic relation in vertex form $y = a(x-p)^2 + q$ can be converted to standard form $y = ax^2 + bx + c$ by *expanding* and collecting like terms.
- A quadratic relation in standard form $y = ax^2 + bx + c$ can be converted to vertex form $y = a(x-p)^2 + q$ by *completing the squares* which will be discussed in this unit.

$y = a(x-p)^2 + q$
$p < 0$ & $q > 0$
$Vertex: (p,q)$
Axis of Symmetry: $x = p$
$a < 0$ Concaves down
Reflects about x-axis



$y = a(x-p)^2 + q$
$p > 0$ & $q < 0$
$Vertex: (p,q)$
Axis of Symmetry: $x = p$
$a > 0$ Concave up

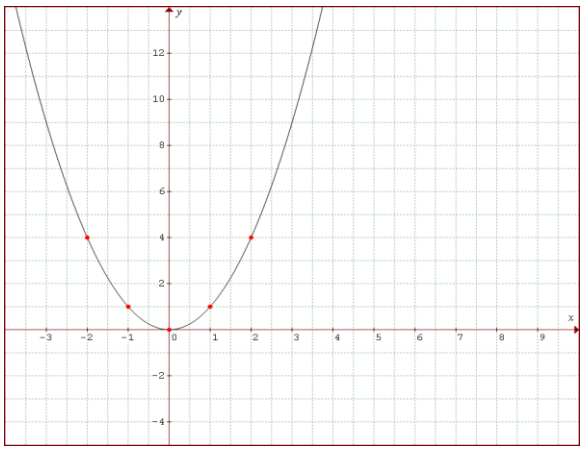
x – intercepts
$x = p \pm \sqrt{\frac{-q}{a}}$

y – intercept
$y = ap^2 + q$

Example 1:
Find the vertex, the axis of symmetry, the direction of opening, x-intercept(s) and the y-intercept for the graph of the quadratic relation. State the mapping rule.

a) $y = \frac{1}{2}(x-5)^2 - 2$:

$y = x^2$		***Mapping rule***	
x	***y***		
-2	***4***		
-1	***1***		
0	***0***		
1	***1***		
2	***4***		



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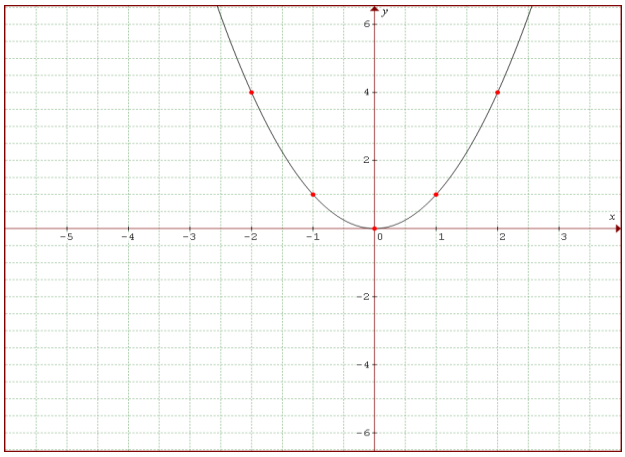
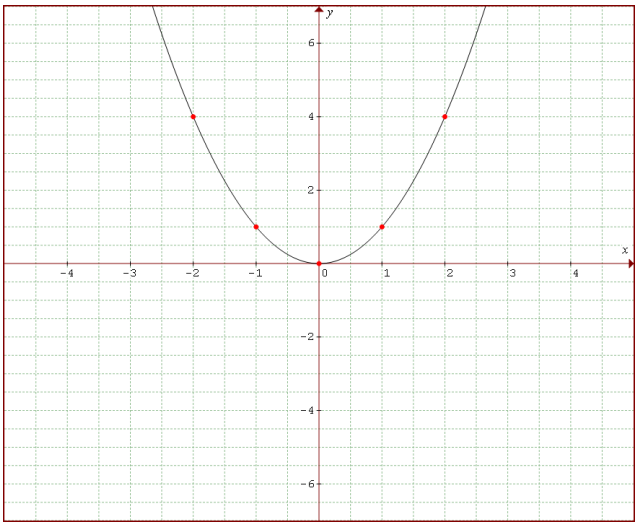
b) $y = -2(x+3)^2 + 4$

$y = x^2$		***Mapping rule***	
x	***y***		
-2	***4***		
-1	***1***		
0	***0***		
1	***1***		
2	***4***		



c) $y = -\frac{1}{3}(x+1)^2 - 2$

$y = x^2$		***Mapping rule***	
x	***y***		
-2	***4***		
-1	***1***		
0	***0***		
1	***1***		
2	***4***		



Example 2:

Determine a quadratic relation in vertex form which contains vertex $(-4,-5)$ and passes through the point $(2,-6)$. Determine the mapping rule for the transformations.

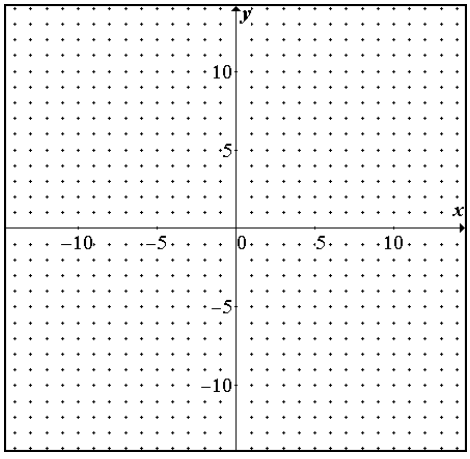
Exercise

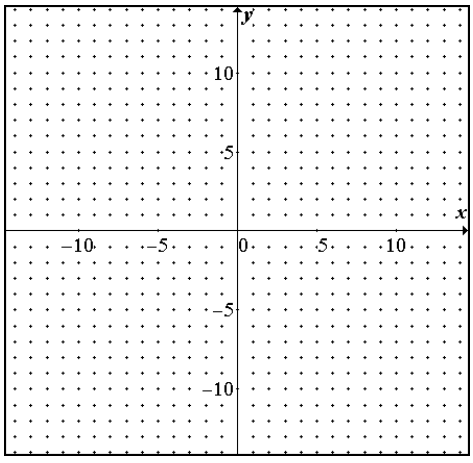
1. Complete the following table:

Equation	vertex	axis of symmetry	Opening	*x*-intercepts	*y*-intercept
a) $y = x^2$					
b) $y = (x-5)^2$					
c) $y = (x-5)^2 + 5$					
d) $y = (x+2)^2$					
e) $y = (x+2)^2 - 4$					

2. Use the table above to sketch the graph of the quadratic relation. State the mapping rule.

$y = x^2$ $y = (x-5)^2$ $y = (x-5)^2 + 5$ $y = (x+2)^2$ $y = (x+2)^2 - 4$





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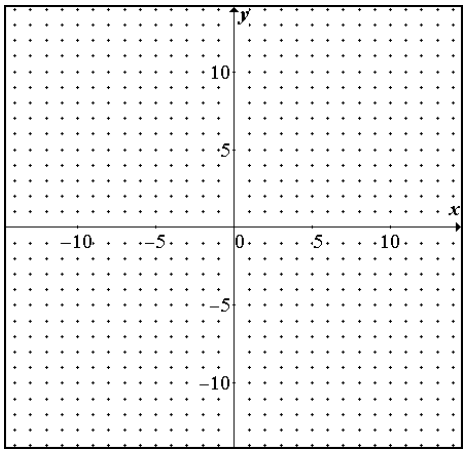
4. Complete the following table:

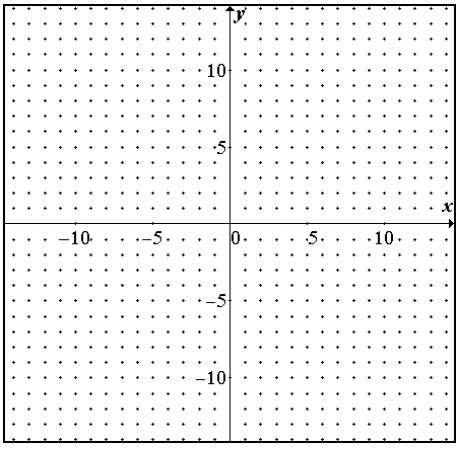
Equation	**vertex**	**axis of symmetry**	**Opening**	***x*-intercepts**	***y*-intercept**
a) $y=(x-3)^2$					
b) $y=(x-3)^2-2$					
c) $y=(x-3)^2+4$					
d) $y=(x-3)^2-4$					

5. Complete the following table:

Equation	**vertex**	**axis of symmetry**	**Opening**	***x*-intercepts**	***y*-intercept**
a) $y=(x-3)^2+4$					
b) $y=(x-1)^2+4$					
c) $y=(x+2)^2+4$					
d) $y=(x+5)^2+4$					

6. Use the tables in #4,5 above to sketch the graphs of the quadratic relation. State the mapping rules.





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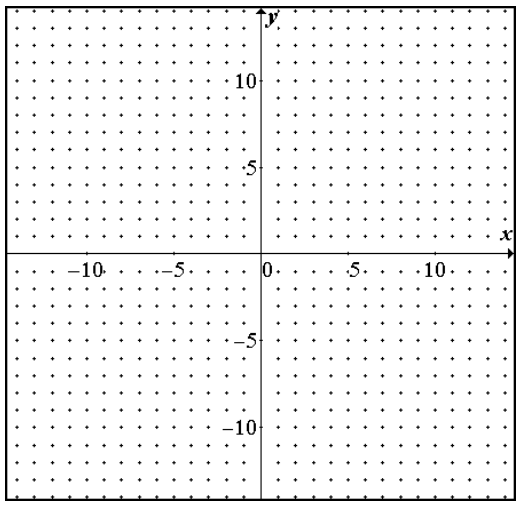
7. Complete the following table:

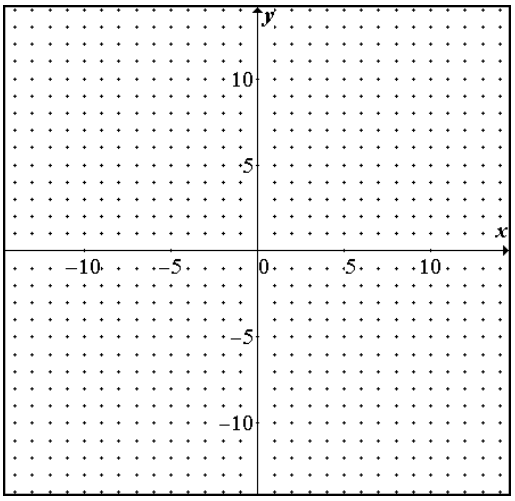
Equation	**vertex**	**axis of symmetry**	**Opening**	***x*-intercepts**	***y*-intercept**
a) $y=(x-2)^2+4$					
b) $y=3(x-2)^2+4$					
c) $y=-(x-2)^2+4$					
d) $y=-3(x-2)^2+4$					

8. Complete the following table:

Equation	**vertex**	**axis of symmetry**	**Opening**	***x*-intercepts**	***y*-intercept**
a) $y=(x+2)^2-3$					
b) $y=\frac{1}{2}(x+2)^2-3$					
c) $y=-\frac{1}{2}(x+2)^2-3$					

9. Use the tables in #7,8 above to sketch the graphs of the quadratic relation. State the mapping rules.





10. In comparing the graph of $y = 2(x-4)^2 + 5$ to the graph of $y = x^2$, explain:

a) what change is caused by the '5'?

_____

b) what change is caused by the '4'?

_____

c) what change is caused by the '2'?

_____

11. Given the relation $y = 2(x-4)^2 + 5$

a) what is the vertex of the parabola? _____

b) does the graph open upwards or downwards? _____

c) is the vertex a maximum or minimum point? _____

The equation $y = a(x-p)^2 + q$ is called the **vertex form** of the parabola.

12. Given the relation $y = a(x-p)^2 + q$

a) what change is caused by the 'q'?

_____

b) what change is caused by the 'p'?

_____

c) what change is caused by the 'a'?

_____

13. Given the relation $y = a(x - p)^2 + q$

a) what is the vertex of the parabola? _____

b) does the graph open upwards or downwards? _____

c) is the vertex a maximum or minimum point? _____

14. Write the corresponding equation in vertex form for the quadratic relation with the given values.

	a	**vertex**	**equation**
a)	2	(1,3)	
b)	2	(1,–3)	
c)	2	(–1,3)	
d)	2	(–1,–3)	
e)	-2	(1,3)	
f)	$-\frac{1}{2}$	(–4,7)	

15. For each of the following:

a) create the general equation in vertex form.

b) use the given point to determine the value of 'a'.

c) write the defining equation.

(i) vertex (3,5)
point on parabola (1,–3)

(ii) vertex (–1,–7)
point on parabola (0,–3)

(iii) vertex (2,–5)
point on parabola (4,–7)

(iv) vertex (4,1)
point on parabola (1,7)

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Answers:

1a) Vertex: (0, 0); Axis of symmetry: $x = 0$; Opens up; x-int: $x = 0$; y – int: $y = 0$;

b) Vertex: (5, 0); Axis of symmetry: $x = 5$; Opens up; x-int: $x = 5$; y – int: $y = 25$; $(x+5, y)$

c) Vertex: (5, 5); Axis of symmetry: $x = 5$; Opens up; x-int: *NA*; y – int: $y = 30$; $(x+5, y+5)$

d) Vertex: (-2, 0); Axis of symmetry: $x = -2$; Opens up; x-int: $x = -2$; y – int: $y = 4$; $(x-2, y)$

e) Vertex: (-2, -4); Axis of symmetry: $x = -2$; Opens up; x-int: $x = -4$ & 0; y – int: $y = 0$; $(x-2, y-4)$

4a) Vertex: (3, 0); Axis of symmetry: $x = 3$; Opens up; x-int: $x = 3$; y – int: $y = 9$; $(x+3, y)$

b) Vertex: (3, -2); Axis of symmetry: $x = 3$; Opens up; x-int: $x = 1.6$ & 4.4; y – int: $y = 7$; $(x+3, y-2)$

c) Vertex: (3, 4); Axis of symmetry: $x = 3$; Opens up; x-int: NA; y – int: $y = 13$; $(x+3, y+4)$

d) Vertex: (3, -4); Axis of symmetry: $x = 3$; Opens up; x-int: $x = 1$ & 5; y – int: $y = 5$; $(x+3, y-4)$

5a) Vertex: (3, 4); Axis of symmetry: $x = 3$; Opens up; x-int: NA; y – int: $y = 13$; $(x+3, y+4)$

b) Vertex: (1, 4); Axis of symmetry: $x = 1$; Opens up; x-int: NA; y – int: $y = 5$; $(x+1, y+4)$

c) Vertex: (-2, 4); Axis of symmetry: $x = -2$; Opens up; x-int: NA; y – int: $y = 8$; $(x-2, y+4)$

d) Vertex: (-5, 4); Axis of symmetry: $x = -5$; Opens up; x-int: NA; y – int: $y = 29$; $(x-5, y+4)$

7a) Vertex: (2, 4); Axis of symmetry: $x = 2$; Opens up; x-int: NA; y – int: $y = 8$; $(x+2, y+4)$

b) Vertex: (2, 4); Axis of symmetry: $x = 2$; Opens up; x-int: NA; y – int: $y = 16$; $(x+2,3y+4)$

c) Vertex: (2, 4); Axis of symmetry: $x = 2$; Opens down; x-int: $x = 0$ & 4; y – int: $y = 0$; $(x+2,-y+4)$

d) Vertex: (2, 4); Axis of symmetry: $x = 2$; Opens down; x-int: $x = 0.8$ & 3.2; y – int: $y = -8$; $(x+2,-3y+4)$

8a) Vertex: (-2, -3); Axis of symmetry: $x = -2$; Opens up; x-int: $x = -3.7$ & -0.3; y – int: $y = 1$; $(x-2, y-3)$

b) Vertex: (-2, -3); Axis of symmetry: $x = -2$; Opens up; x-int: $x = -4.4$ & 0.4; y – int: $y = -1$; $\left(x-2, \frac{1}{2}y-3\right)$

c) Vertex: (-2, -3); Axis of symmetry: $x = -2$; Opens down; x-int: NA; y – int: $y = -5$; $\left(x-2, \frac{-1}{2}y-3\right)$

10a) Vertical translation by 5 units up b) Horizontal translation by 4 units right c) Vertical expansion by factor of 2

11a) (4, 5) b) upward c) minimum

12a) Vertical translation by q units (if $q > 0$, up. $q < 0$, down)

b) Horizontal translation by p units (if $p > 0$, Right by p units, $p < 0$, Left by p units)

c) Vertical stretch by factor of a (if $a > 1$, expand, $0 < a < 1$, compress) (if $a > 0$, opens up, $a < 0$, opens down)

13a) (p, q) b) if $a > 0$, opens up, $a < 0$, opens down c) if $a > 0$, min, $a < 0$, max

14a) $y = 2(x-1)^2 + 3$ b) $y = 2(x-1)^2 - 3$ c) $y = 2(x+1)^2 + 3$

d) $y = 2(x+1)^2 - 3$ e) $y = -2(x-1)^2 + 3$ f) $y = \frac{-1}{2}(x+4)^2 + 7$

15i) a) $y = a(x-3)^2 + 5$ b) -2 c) $y = -2(x-3)^2 + 5$ ii) a) $y = a(x+1)^2 - 7$ b) 4 c) $y = 4(x+1)^2 - 7$

iii) a) $y = a(x-2)^2 - 5$ b) $\frac{-1}{2}$ c) $y = \frac{-1}{2}(x-2)^2 - 5$ v) a) $y = a(x-4)^2 + 1$ b) $\frac{2}{3}$ c) $y = \frac{2}{3}(x-4)^2 + 1$