Perfect Square Trinomials and Completing The Square

Date:

A quadratic relation in standard form $y = ax^2 + bx + c$ can be rewritten in vertex form $y = a(x - p)^2 + q$ by creating a perfect square in the expression, then factoring the square. This technique is called **completing the square.**

- Completing the square involves the following steps:
  1) Remove the common constant factor from both the x^2 and x term.
  2) Find the constant that must be added and subtracted to create a **perfect square**. This value equals the square of half of the coefficient of the x –term in step 1. Rewrite the expression by adding, then subtracting, this value after the x – term inside the brackets.
  3) Group the three terms that form the perfect square. Move the subtracted value outside the brackets by multiplying it by the common constant factor.
  4) Factor the perfect square and collect like terms.
- Completing the square can be used to find the vertex of a quadratic in standard form without finding the zeros of the relation or two points equidistant from the axis of summetry.
- Completing the square allows you to find the maximum or minimum value of a quadratic relation algebraically, without using a graph.

Example 1:

Convert each of the following into *vertex form* by **completing the square.**

a) $y = x^2 + 6x$ b) $y = x^2 + 8x - 3$ c) $y = x^2 + 5x + 2$

Example 2:

Convert each of the following into *vertex form* by **completing the square.**

Determine the vertex, axis of symmetry, opening, intercepts and the mapping rule.

a) $y = -x^2 - 4x + 5$ b) $y = 3x^2 + 12x - 5$

c) $y = 2x^2 + 5x - 1$ TRY: $y = ax^2 + bx + c$

Exercise

1. Expand each of the following, giving *final answers* only. Do not show a middle step.

a) $(x+5)^2$ *b*) $(x-4)^2$

c) $(x+1)^2$ *d*) $(x+7)^2$

e) $(x-\frac{1}{3})^2$ *f*) $(x+\frac{3}{4})^2$

Each of the answers in #1 is called a <u>perfect square trinomial</u> because it is the result of squaring a binomial.

Expand $(x+10)^2$ and $(x+10)(x-10)$.

In what way are the answers different? _____

In what ways are the answers the same? _____

2. Which of the following trinomials are *perfect square trinomials?* (circle the letter)

(try to discover a pattern from the answers above)

a) x^2+6x+9 *b*) x^2+6x-9 *c*) x^2-6x+9 *d*) x^2-6x-9

e) $x^2+8x+64$ *f*) $x^2+36x+64$ *g*) $x^2+16x+64$ *h*) $x^2-16x+64$

i) $x^2+9x+81$ *j*) $x^2-9x+16$ *k*) $x^2+9x+\frac{81}{4}$ *l*) $x^2+9x-\frac{81}{4}$

m) $x^2+25x+10$ *n*) x^2+x+1

3. Fill in the blank with the appropriate number to make each a perfect square trinomial.

a) x^2+12x _____ *b*) x^2-20x_____

c) x^2-10x_____ *d*) x^2+4x_____

e) x^2+_____$+9$ *f*) x^2-_____$+4$

g) x^2-_____$+1$ *h*) x^2+_____$+121$

Perfect Square Trinomials and Completing The Square Date:

4. Beside each of the trinomials in #3, write its factored form. Express your answer as $(x+p)^2$ or $(x-p)^2$.

5. Convert each of the following into *vertex form* by **completing the square.**

a) $y=x^2+8x$ *b*) $y=x^2-10x$ *c*) $y=x^2+18x$

d) $y=x^2+16x-4$ *e*) $y=x^2-12x-5$ *f*) $y=x^2+10x+1$

g) $y=x^2+7x+3$ *h*) $y=x^2+7x+1$ *i*) $y=x^2-3x-5$

6. Without graphing each function, state whether it has a maximum or a minimum value. State the maximum or minimum value of the function. State the value of *x* when it occurs.

a) $y=x^2+6x+2$ *b*) $y=3x^2+6x-8$

c) $y=2x^2-4x+5$ *d*) $y=-2x^2-12x$

e) $y=4-6x-x^2$ *f*) $y=2x^2+3x+3$

g) $y=-4x^2+8x-4$ *h*) $y=4x^2-16x$

i) $y=-28+10x-x^2$ *j*) $y=x^2-12x+36$

7. Complete the chart.

Standard form	**Vertex form**	**Direction of opening**	**Equation of axis of symmetry**	**Vertex**	**Is vertex a minimum or a maximum point?**	**Minimum or maximum value**	***y*-intercept**
$y=x^2-8x+3$							
$y=-2x^2+12x-12$							
$y=x^2-3x+5$							
$y=3x^2-12x+5$							
	$y=3(x-1)^2-7$						
	$y=-(x-3)^2+5$						

Perfect Square Trinomials and Completing The Square Date:

Answers

1*a*) $x^2+10x+25$ *b*) $x^2-8x+16$ *c*) x^2+2x+1 *d*) $x^2+14x+49$ *e*) $x^2-\frac{2}{3}x+\frac{1}{9}$ *f*) $x^2+\frac{3}{2}x+\frac{9}{16}$

2) a c g h k $c>0,\ c=\left(\frac{1}{2}b\right)^2$

3a) 36 b) 100 c) 25 d) 4 e) $6x$ f) $4x$ g) $2x$ h) $22x$

4a) $(x+6)^2$ b) $(x-10)^2$ c) $(x-5)^2$ d) $(x+2)^2$ e) $(x+3)^2$ f) $(x-2)^2$ g) $(x-1)^2$ h) $(x+11)^2$

5a) $y=(x+4)^2-16$ b) $y=(x-5)^2-25$ c) $y=(x+9)^2-81$ d) $y=(x+8)^2-68$

e) $y=(x-6)^2-41$ f) $y=(x+5)^2-24$ g) $y=\left(x+\frac{7}{2}\right)^2-\frac{37}{4}$ h) $y=\left(x+\frac{7}{2}\right)^2-\frac{45}{4}$ i) $y=\left(x-\frac{3}{2}\right)^2-\frac{29}{4}$

6a) Min value = -7 when x = -3 b) Min value = -11 when x = -1 c) Min value = 3 when x = 1

d) Max value = 18 when x = -3 e) Max value = 13 when x = -3 f) Min value = $\frac{15}{8}$ when $x=\frac{-3}{4}$

g) Max value = 0 when x = 1 h) Min Value = -16 when x = 2 i) Max value = -3 when x = 5

j) Min value = 0 when x = 6

7)

Standard form	**Vertex form**	**Direction of opening**	**Equation of axis of symmetry**	**Vertex**	**Is vertex a minimum or a maximum point?**	**Minimum or Maximum value**	***y*-intercept**
$y=x^2-8x+3$	$y=(x-4)^2-13$	Up	x = 4	(4, -13)	Min	-13	3
$y=-2x^2+12x-12$	$y=-2(x-3)^2+6$	Down	x = 3	(3, 6)	Max	6	-12
$y=x^2-3x+5$	$y=(x-\frac{3}{2})^2+\frac{11}{4}$	Up	x = 1.5	$\left(\frac{3}{2},\frac{11}{4}\right)$	Min	$\frac{11}{4}$	5
$y=3x^2-12x+5$	$y=3(x-2)^2-7$	Up	x = 2	(2, -7)	Min	-7	5
$y=3x^2-6x-4$	$y=3(x-1)^2-7$	Up	x = 1	(1, -7)	Min	-7	-4
$y=-x^2+6x-4$	$y=-(x-3)^2+5$	Down	x = 3	(3,5)	Max	5	-4