Quadratic Formula

Date:

Recall: Convert the Quadratic relations from Standard form to Vertex form.

$$y = ax^2 + bx + c \quad \text{(Standard Form)}$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 + 4ac}{4a}$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) \quad \text{(Vertex Form)}$$

$$Vertex: \left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right) or \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

To find the roots, let $y = 0$ and this will derive the Quadratic Formula

$$ax^2 + bx + c = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(Quadratic Formula!!)}$$

Example 1: Solving Quadratic equation using the quadratic formula

Solve each of the following using the quadratic formula and leave the answer in square root form.

a) $x^2 + 7x + 2 = 0$ b) $x^2 - x - 2 = 0$ c) $-2x^2 - x + 5 = 0$

Example 2:

Find the roots of the following using the quadratic formula and accurate the answer in two decimal places.

a) $y = 3x^2 - x - 1$ b) $y = 2x^2 + 3x - 7$ c) $y = -5x^2 + 14x + 3$

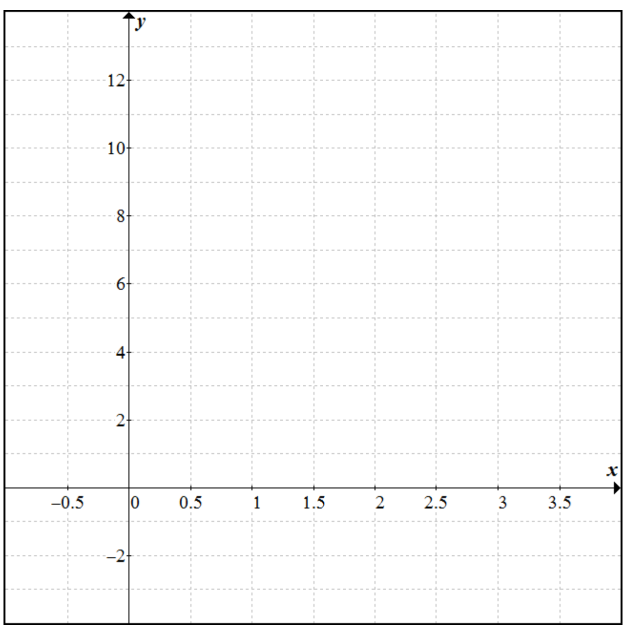
Example 3:

The height of an object, in metres, is given by the relation $h = 15 + 28t - 4.9t^2$ where t is in seconds. How long is the object in the air?

Example 4:

For the quadratic relation $y = -5x^2 + 14x + 3$ (Example 2c)

a. Convert the defining equation of the relation from standard form into vertex form.
b. Use the quadratic formula to approximate the roots.
c. Determine the vertex, axis of symmetry and y – intercept.
d. Sketch the graph using the key the results from b & c
e. State the mapping rule.



Quadratic Formula

Date:

Exercise:

1) Solve each equation with the Quadratic Formula.

a) $2x^2+x-21=0$ b) $x^2+2x-3=0$ c) $2x^2+x-15=0$

d) $x^2-4x-21=0$ e) $2x^2-3x-20=0$ f) $2x^2+x-1=0$

g) $5x^2-15x-24=4x^2-5x$ h) $11x^2+12x=5x^2-6$

i) $13x^2-12x-85=12x^2$ j) $4x^2-8=4x$

k) $x^2-3x=40$ l) $x^2-7x=5x+28$ m) $2x^2-33=3x+2$

2) Solve each of the following using the quadratic formula and leave the answer in square root form.

a) $8x^2-x-19=-9x$ b) $4x^2+8x-11=-2$ c) $-4x^2+4x+3=-3$

3) For each of the following

a. Convert the defining equation of the relation from standard form into vertex form.
b. Use the quadratic formula to approximate the roots.
c. Determine the vertex, axis of symmetry and *y* – intercept.
d. Sketch the graph using the key the results from b & c

i) $y=x^2-5x-1$ ii) $y=x^2+2x+6$

iii) $y=2x^2+16x+32$ iv) $y=-3x^2+12x+6$

Answers:

1a) 3 & -3.5 b) 1 & -3 c) 2.5 & -3 d) 7 & -3 e) 4 & -2.5 f) 0.5 & -1 g) 12 & -2 h) -1
i) 17 & -5 j) 2 & -1 k) 8 & -5 l) 14 & -2 m) 5 & -3.5

2a) $\dfrac{-2\pm\sqrt{42}}{4}$ b) $\dfrac{-2\pm\sqrt{13}}{2}$ c) $\dfrac{1\pm\sqrt{7}}{2}$

3i) $y=(x-2.5)^2-7.25$ Roots: -0.2 & 5.2 Vertex: (2.5, -7.25) Axis of symmetry: *x* = 2.5 *y*-int: -1

ii) $y=(x+1)^2+5$ Roots: None Vertex: (-1, 5) Axis of symmetry: *x* = -1 *y*-int: 6

iii) $y=2(x+4)^2$ Roots: -4 Vertex: (-4, 0) Axis of symmetry: *x* = -4 *y*-int: 32

iv) $y=-3(x-2)^2+18$ Roots: $2\pm\sqrt{6}$ Vertex: (2, 18) Axis of symmetry: *x* = 2 *y*-int: 6