Optimization Problems (More)

Date:

Recall: Convert the Quadratic relations from Standard form to Vertex form.

$$y = ax^2 + bx + c \quad \text{(Standard Form)}$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 + 4ac}{4a}$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) \quad \text{(Vertex Form)}$$

$$Vertex: \left(-\frac{b}{2a}, -\frac{b^2 - 4ac}{4a}\right) or \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

To find the roots, let $y = 0$ and this will derive the Quadratic Formula

$$ax^2 + bx + c = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right) = 0$$
$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(Quadratic Formula!!)}$$

Example 1: Determine the Vertex from Standard Form

i) Convert the following quadratic relations to vertex form to determine the vertex.
ii) Use the short cut (vertex formula) to determine the vertex from standard form.

a) $y = 3x^2 + 12x - 15$

b) $y = -2x^2 + 3x - 8$

Example 2: Optimization Problems

Find two real numbers *x* and *y* that sum to 50 and that have a product that is a maximum.

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Example 3: Optimization Problems

Last year the yearbook at Central High cost $75 and only 500 were sold. A student survey found that for every $5 reduction in price, 100 more students will buy yearbooks. What price should be charged to maximize the revenue from yearbook sales?

Example 4: Optimization Problems

Mary wants to fence a rectangular garden to keep the deer from eating her fruit and vegetables. One side of her garden attaches her shed wall so she will not need to fence that side. However, she also wants to use material to separate the rectangular garden in two sections. She can afford to buy 80 total feet of fencing to use for the perimeter and the section dividing the rectangular garden. What dimensions will maximize the total area of the rectangular garden?

Example 5: Optimization Problems

A piece of wire 20 feet long is cut into two pieces and each piece is bent to form a square. Determine the length of the two pieces so that the sum of the areas of the two squares is a minimum.