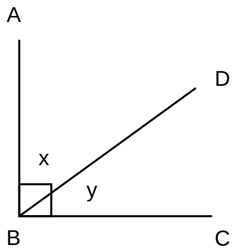
Basic Angle Theorems

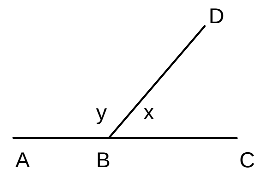
Complementary Angles (CA)



If AB ⊥ BC, then
$\angle ABD + \angle DBC = 90°$

or, $x + y = 90°$

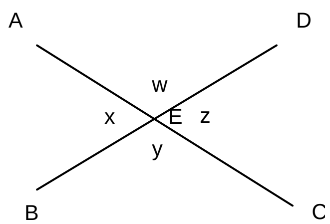
Supplementary Angles (SA)



If $\angle ABC$ is a straight line, then
$\angle CBD + \angle ABD = 180°$

or $x + y = 180°$

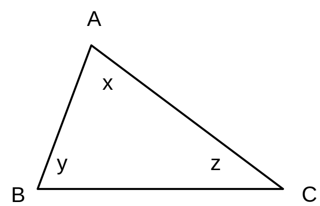
Opposite Angle Theorem (OAT)



If AC and BD are line segments intersecting at E, then, $x = z$

and, $w = y$

Sum of Angles in a Triangle Theorem (SATT)

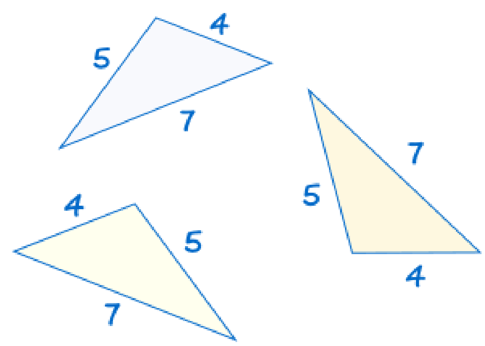


In any triangle, the sum of the angles is 180^{o}
That is:

$x + y + z = 180^{o}$

Congruent Triangles

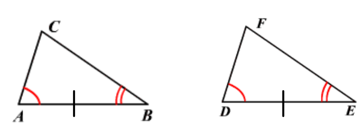
When two triangles are **congruent** they will have exactly **the same three sides** and exactly **the same three angles**. The equal sides and angles may not be in the same position (if there is a turn or a flip), but they are there.



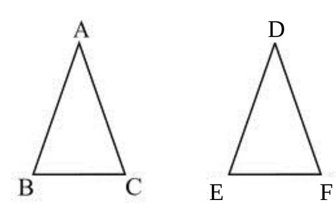
How to tell if triangles are congruent

Any triangle is defined by six measures (three sides, three angles). But you don't need to know all of them to show that two triangles are congruent. Various groups of three will do. Triangles are congruent if:

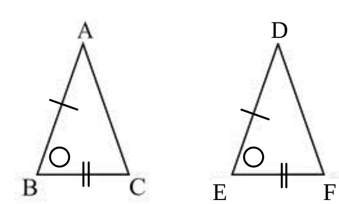
1. **ASA** (angle side angle)
   - Two pairs of corresponding angles and the *contained sides* are equal.
     Given: $\angle A = \angle D$, $\angle B = \angle E$, $AB = DE$
     Conclusion: $\Delta ABC \cong \Delta DEF$



2. **SSS in same proportion** (side side side)
   - All three pairs of corresponding sides are equal.
     Given: $AB = DE, BC = EF, AC = DF$
     Conclusion: $\Delta ABC \cong \Delta DEF$



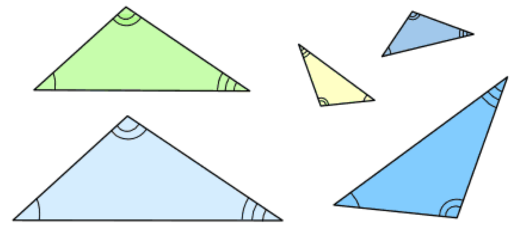
3. **SAS (side angle side)**
   - Two pairs of corresponding sides and the *contained angles* are equal.
     Given: $AB = DE, \angle B = \angle E$, $BC = EF$
     Conclusion: $\Delta ABC \cong \Delta DEF$



Similar Triangles

Two triangles are **similar** if the only difference is size (and possibly the need to turn or flip one around).

- Equal angles have been marked with the same label or same number of arcs)
- Some of them have different sizes and some of them have been turned or flipped.
- Similar triangles have all their angles equal and their corresponding sides have the same ratio.



How to tell if triangles are similar

Any triangle is defined by six measures (three sides, three angles). But you don't need to know all of them to show that two triangles are similar. Various groups of three will do. Triangles are similar if:

1. AAA or **AA~** (angle angle angle)

- At least two pairs of corresponding angles are the same.

Given: $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

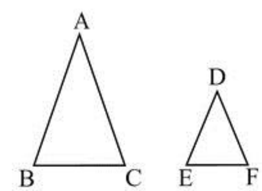
Conclusion: $\Delta ABC \sim \Delta DEF$

2. SSS in same proportion (side side side)

- All three pairs of corresponding sides are in the same proportion

Given: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Conclusion: $\Delta ABC \sim \Delta DEF$

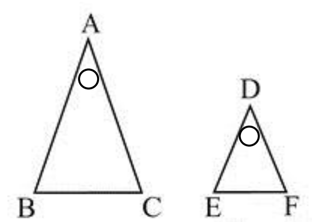


3. SAS (side angle side)

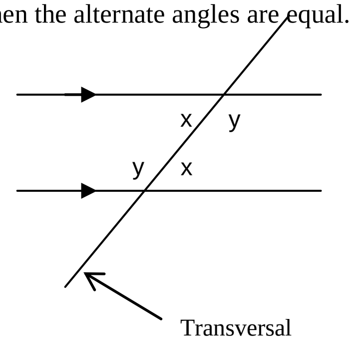
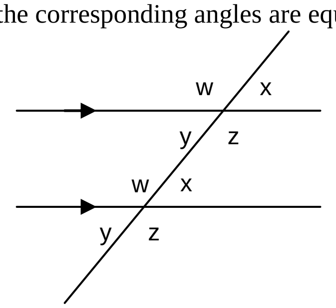
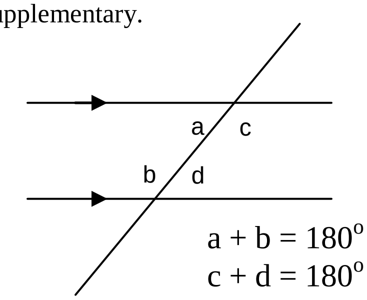
- Two pairs of corresponding sides in the same proportion and the *contained angles* are equal.

Given: $\angle A = \angle D, \frac{AB}{DE} = \frac{AC}{DF}$

Conclusion: $\Delta ABC \sim \Delta DEF$



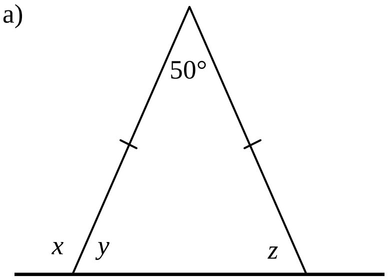
Parallel Line Theorems (PLT)

Alternate Angle Theorem (PLT- Z)	**Corresponding Angle Theorem (PLT- F)**	**Interior Angle Theorem (PLT - C)**
If a transversal intersects two parallel lines, then the alternate angles are equal.	If a transversal intersects two parallel lines, then the corresponding angles are equal.	If a transversal intersects two parallel lines, then the interior angles are supplementary.
		

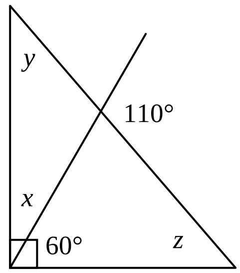
Example 1:

Determine the unknown measures

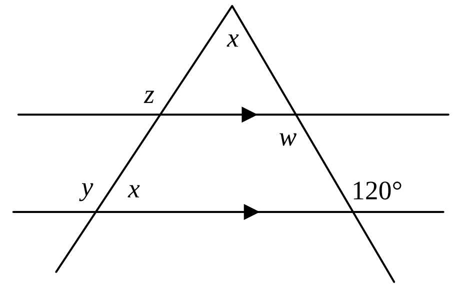
a)



b)

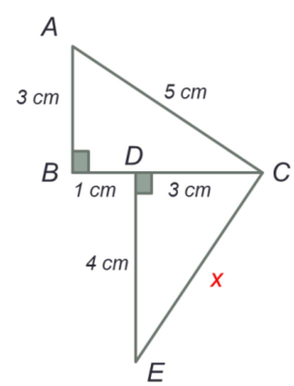


c)



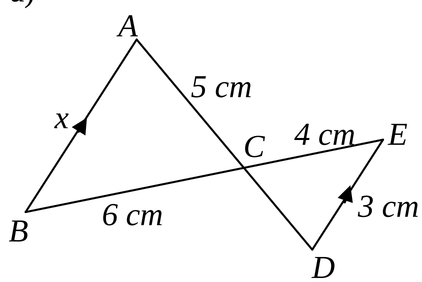
Example 2:

Verify the congruent triangles and determine the unknown measures

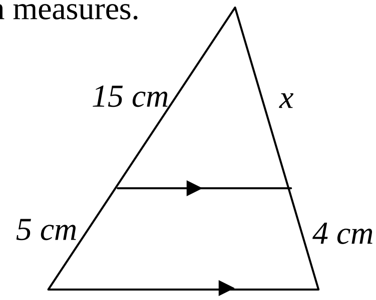


Example 3:
Verify the similar triangles and determine the unknown measures.

a)

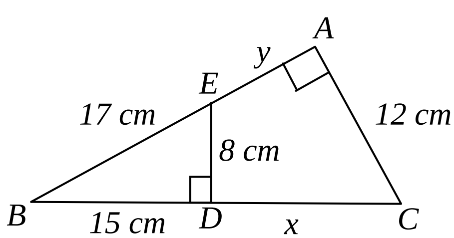


b)



What if the triangles are not labelled?

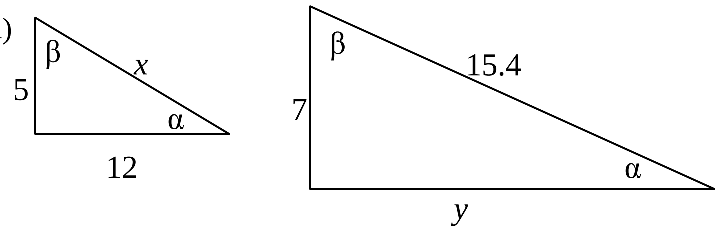
Example 4:
Determine the unknown measures.



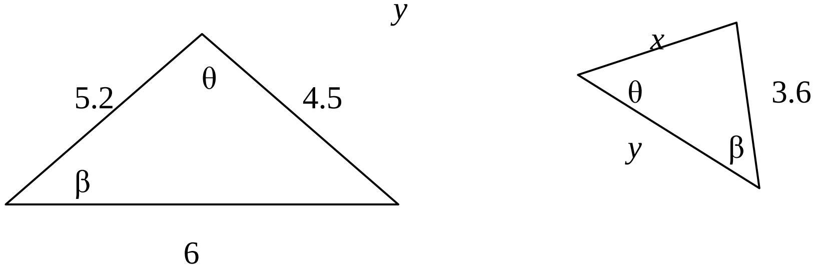
Exercise

1. Determine the values of x and y.

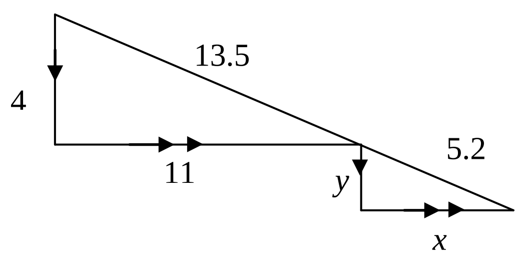
a)



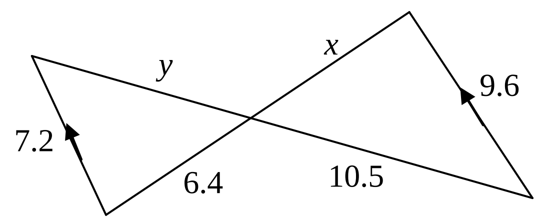
b)



c)



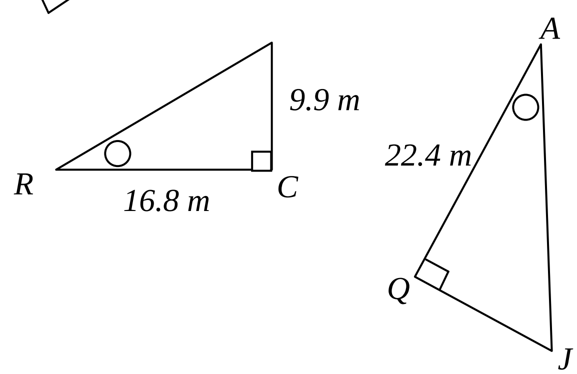
d)



2. In ΔMRC and ΔJAQ, $\angle R = \angle A$ and $\angle C = \angle Q = 90°$.

a) Explain why ΔMRC and ΔJAQ are similar.

b) Use the dimensions given on the diagram to find the length of JQ.



Answers:

1a) 11; 16.8 b) 3.12; 2.7 c) $\frac{572}{135};\frac{208}{135}$ d) $\frac{128}{15};\frac{63}{8}$

2a) Since $\angle R = \angle A$ and $\angle C = \angle Q$ and the sum of the angles in a triangle is $180°$, $\angle M = \angle J$. Therefore $\Delta MRC \sim \Delta JAQ$. *(AA~)* b) 13.2 m