Velocity and Other Rates of Change

Date:

Motion on a straight Line

An object that moves along a straight line with its position determined by a function of time, $s(t)$, has a velocity of $v(t) = s'(t)$ and an acceleration of $a(t) = v'(t) = s''(t)$ at time *t*.

In Leibniz notation, $v = \frac{ds}{dt}$ $\quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

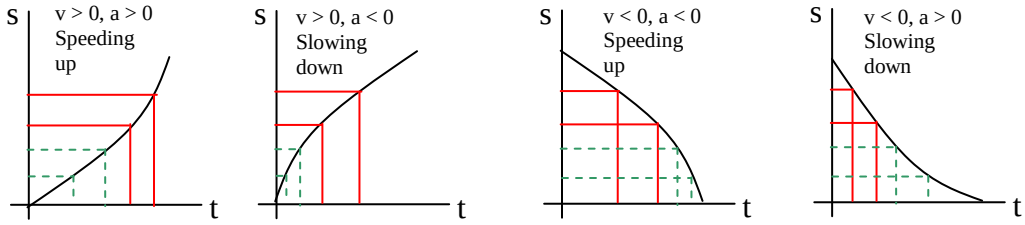
About Velocity

- If $v(t) > 0$, object is moving in a positive directions (right or up) at time *t*;
- If $v(t) < 0$, object is moving in a negative directions (left or down) at time *t*;
- If $v(t) = 0$, object is not moving and a change in direction may occur at time *t*.

About Acceleration

- If $a(t) > 0$, object's velocity is increasing;
- If $a(t) < 0$, object's velocity is decreasing;
- If $a(t) = 0$, object's velocity is constant and steady.

Object is speeding up (accelerating) if $v(t)\,a(t) > 0$.
Object is slowing down (decelerating) if $v(t)\,a(t) < 0$.



Example 1: Motion to Displacement and Velocity

The position of a particle moving on a line is given by the equation $s(t) = 2t^3 - 21t^2 + 60t,\ t \geq 0$, where *t* is measured in seconds and *s* in metres.

a) What is the velocity after 2 sec and after 6 sec?
b) When is the particle at rest?
c) When is the particle moving in the positive direction/negative direction?
d) Find the total distance travelled by the particle during the first 6 seconds.

Example 2: Velocity and Acceleration

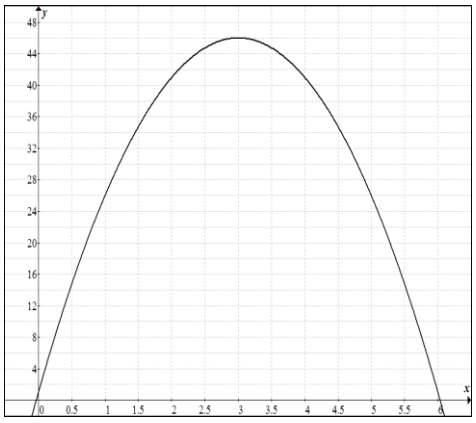
A particle moves according to the equation of motion $s(t) = t^3 - 9t^2 + 18t,$ where t is measured in seconds and s in metres.

a) When is the acceleration 0?
b) Find the displacement and velocity at that time.
c) At $t = 4$, is the object speeding up or slowing down at $t = 4$?

Example 3: Analyzing motion under gravity near surface of Earth

A baseball is hit vertically upward. The position function $s(t)$, in metres, of the ball above the ground is $s(t) = -5t^2 + 30t + 1,$ where t is in seconds.

a) Determine the maximum height reached by the ball.
b) Determine the velocity of the ball when it is caught 1 m above the ground.
c) When will the ball be 26 m about the ground?



Marginal Cost, Marginal Revenue and Marginal Profit

In economics and finance, **marginal cost** is the change in total cost that arises when the quantity produced changes by one unit. That is, it is the cost of producing one more unit of a good. **Marginal revenue** is the additional revenue that will be generated by increasing product sales by 1 unit. **Marginal profit** is the term used to refer to the difference between the marginal cost and the marginal revenue for producing one additional unit of production. Profit maximization requires that a firm produce where ***marginal revenue equals marginal costs***.

Example 4: Marginal Cost and Marginal Revenue

Suppose it costs $c(x) = x^3 - 6x^2 + 15x$ dollars to produce x radiators when 8 to 10 radiators are produced, and that $r(x) = x^3 - 3x^2 + 12x$ gives the dollar revenue from selling x radiators. Your shop currently produces 10 radiators a day. Find the **marginal cost** and **marginal revenue**.

Motion under the Influence of Gravity (Galileo Formulas)

Galileo discovered that the height $s(t)$ and velocity $v(t)$ of an object tossed vertically in the air are given as functions of time by the formulas: $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$, $v(t) = \frac{ds}{dt} = v_0 - gt$

- The constants $s_0 = s(0)$ is the position at time $t = 0$.
- $v_0 = v(0)$ is the velocity at $t = 0$.
- $-g$ is the acceleration due to gravity on the surface of the earth (negative because the up direction is positive), where $g \approx 9.8 m/s^2$

Example 5: Finding Initial Conditions

A bullet is fired vertically from an initial height $s_0 = 0$. What initial velocity v_0 is required for the bullet to reach a maximum height of 3 km (3000m) ?

Homework:
P. 135 ##1,3,4,8,9,11,13-16,19-25,28,29
Cal & Vectors (Optional)
P. 127 # 3, 4, 6bc, 8, 10, 11, 12, 14, 15, 17