The Derivatives of Composite Functions (Chain Rule)

Date:

Chain Rule:

If both *f* and *g* are differentiable, then the composite function $h(x) = f\left(g(x)\right)$ has a derivative given by $h'(x) = f'\left(g(x)\right)g'\left(x\right)$.

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, provided that $\frac{dy}{du}$ and $\frac{du}{dx}$ exist.

(Proof can be found in the text book)

Recall: Product Rule

If $p(x) = f(x)g(x)$,

$p'(x) = f'(x)g(x) + f(x)g'(x)$.

Recall: Quotient Rule

$p(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

$p'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$.

Example 1: Using the Chain Rule

Use the Chain rule to differentiate and simplify if

i) $y = \left(4x^2 - 5x + 2\right)^7$

ii) $h(x) = \sqrt{\left(3x^2 - 2\right)^3}$

iii) $y = \frac{-5}{\left(6x^2 - 7\right)^4}$

Example 2: Applying the Chain rule using Leibniz notation

If $y = u^3 - 2u + 1, u = 2\sqrt{x}$, find $\frac{dy}{dx}$ at $x = 4$.

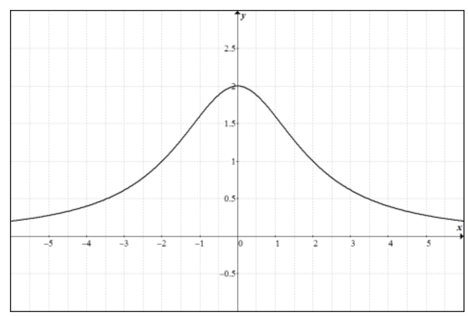
<u>Example 3: Applying the Chain rule with other derivative rules</u>

Differentiate

i) $s(x) = \left(\dfrac{2x-1}{x+2}\right)^6$ ii) $f(x) = (x^2-1)^4\sqrt{(3x^2+7)^3}$

<u>Example 4: Equation of Tangent line</u>

Find the equation of the tangent at the point (2, 1) for $f(x) = \dfrac{8}{x^2+4}$



Homework:
AP: (Save for Chain Rule 2)
<u>Cal & Vectors</u>
P. 105 #1bdef, 4, 5, 7 – 17