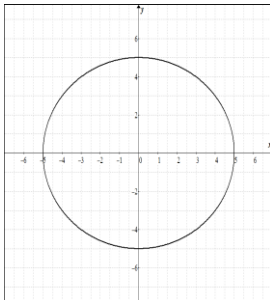
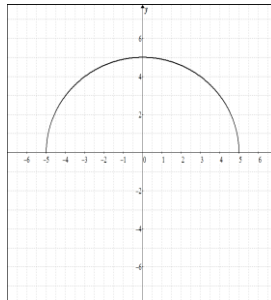


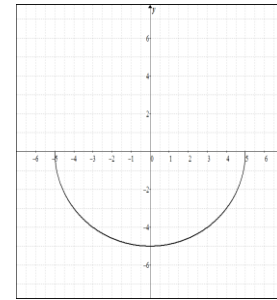
If the functions are expressing one variable **explicitly** in terms of another variable; for example, $y = x^2$ or $y = \frac{\sqrt{4-x^2}}{x+1}$, in general, $y = f(x)$. But other functions are defined **implicitly** by a relation between x and y such as $x^2 + y^2 = 25$. In this case it is possible to solve the equation for y to get $y = \pm\sqrt{25-x^2}$ and so two functions defined by the implicit equations are $f(x) = \sqrt{25-x^2}$ and $g(x) = -\sqrt{25-x^2}$. The graphs of f and g are the upper and lower semicircles of the circle $x^2 + y^2 = 25$.



$$x^2 + y^2 = 25$$



$$f(x) = \sqrt{25-x^2}$$



$$g(x) = -\sqrt{25-x^2}$$

Consider the problem of determining the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$. Since this point lies on the lower circle, we could differentiate the function $g(x) = -\sqrt{25-x^2}$ and substitute $x = 3$. An alternative, which avoids having to solve for y explicitly in terms of x , is to use the method of **implicit differentiation** which involves with the following two simple steps.

Step 1: Differentiate both sides **respected to x** .

Step 2: Solve for $\frac{dy}{dx}$

$$\frac{d}{dx}(\text{LEFT}) = \frac{d}{dx}(\text{RIGHT})$$

Example 1: Differentiations to the Respected Variable

Differentiate the following functions respected to the given variable

a) $y = 4x^3 + 5x$, find $\frac{dy}{dx}$

b) $y = 4a^3 + 5a$, find $\frac{dy}{da}$

c) $y = 4a^3 + 5x$, find $\frac{dy}{dx}$

d) $y = 4x^3 + 5a$, find $\frac{dy}{dx}$

Example 2: Differentiate an Implicit relation

- a) If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.
- b) Find the equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $x = 3$.

Example 3: Using Implicit differentiation to determine derivative

- a) Find $\frac{dy}{dx}$ if $2x^5 + x^4y + y^5 = 36$
- b) Find the slope of the tangent to the curve $2x^5 + x^4y + y^5 = 36$ at the point $(1, 2)$.

Example 4: Using Implicit differentiation to determine derivative

Find g' or $\frac{dg}{da}$ if $a^2 + \sqrt{g} = a^2g^3 + 5$

Example 5: Implicit differentiations with trigonometric functions

If $\tan y = x^2$, find the derivative of y with respect to x .

Example 6: Higher-Order derivative

If $x^3 + y^3 = 5$, find y'' .

Homework:
P. 162 #1-42
Cal & Vectors (Optional)
P. 564 #2, 3cd, 4 - 10,12,13