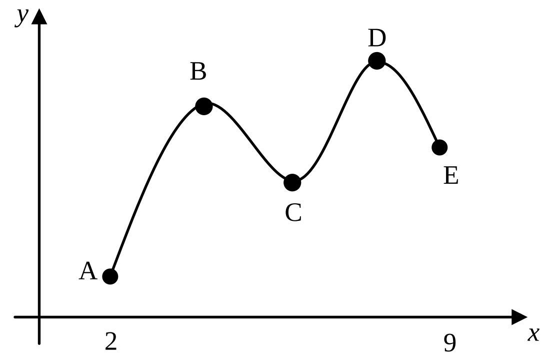
Extreme Values of Functions (First Derivative Test)

Date:

Absolute maximum and minimum points

- The highest and the lowest points in the figure.
- The highest and the lowest points in the domain.
- In the figure, D = absolute max. point; A = absolute min. point
- Also known as Global extreme values.



Local maximum and minimum points

- The turning points.
- The highest and the lowest points in a particular interval.
- In the figure, B & D = local max. points; C = local min. point
- Also known as Relative extreme values.

End points

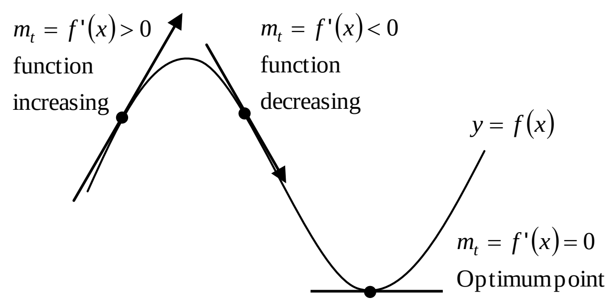
- The extreme points in the figure, usually given in the form $x_1 \le x \le x_2$.
- In the figure, A & E = extreme points ($2 \le x \le 9$).

Critical numbers of *f '*(*x*)

- They are the values of *x* when
  i) $f'(x) = 0$, or ii) $f'(x)$ does not exist (usually we set the denominator of *f'*(*x*)=0 and solve for *x*)
- Critical numbers generate the *x*-coordinates of the turning points.

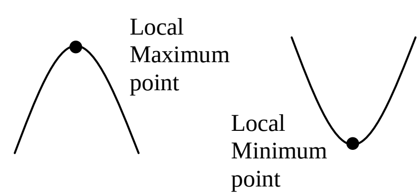
General rules (First Derivative Test)

1) The function is increasing if $f'(x) > 0$.
   The function is decreasing if $f'(x) < 0$.



2) The critical numbers are values of *x* when
   i) $f'(x) = 0$, or
   ii) $f'(x)$ DNE (usually we set the denominator of $f'(x) = 0$.

3) The Local maximum/minimum points are turning points occurring between increasing and decreasing of function or vice versa.



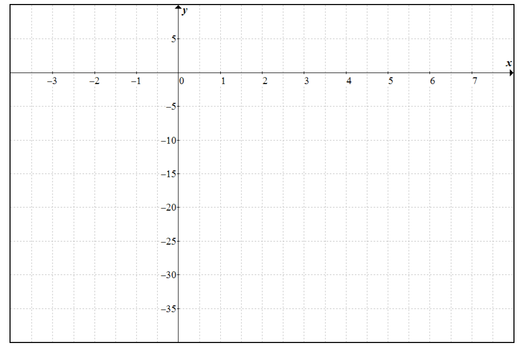
Example 1: Extreme values of a function

Find the extreme values of the function $f(x) = x^3 - 12x - 3$ on the interval $-3 \le x \le 5$.

Example 2: Increasing and decreasing function & its local extreme values

Given $f(x) = x^3 - 6x^2$.

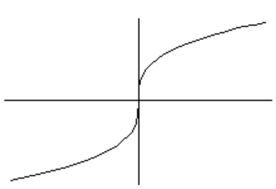
a) Find the critical numbers.
b) Find the intervals of increase and decrease.
c) Find the local maximum and local minimum points.
d) Sketch the graph



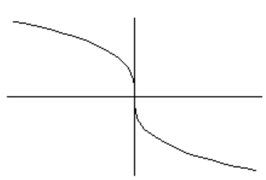
Vertical Tangents and Cusps

There are four possibilities for unbounded behavior of a derivative $f'(x)$ around a given real number ***C***, all of them occur when the critical numbers *x* are obtained from $f'(x)$ **DNE.** The four possible cases are:

Cases 1 & 2: Vertical Tangents (limits same in sign)

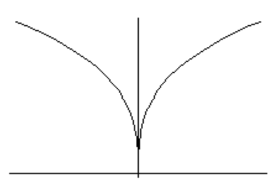


$\lim_{x \to C^-} f'(x) = \lim_{x \to C^+} f'(x) = +\infty$

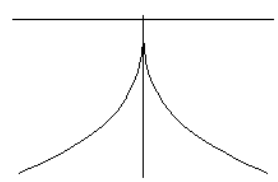


$\lim_{x \to C^-} f'(x) = \lim_{x \to C^+} f'(x) = -\infty$

Cases 3 & 4: Cusps (limits differ in sign)



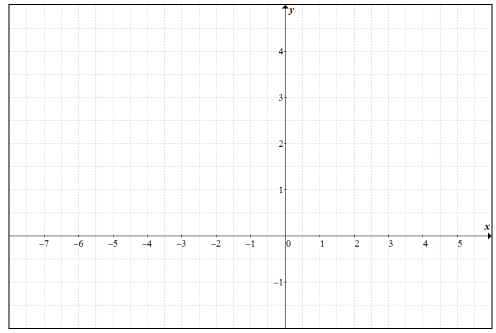
$\lim_{x \to C^-} f'(x) = -\infty$ $\lim_{x \to C^+} f'(x) = +\infty$



$\lim_{x \to C^-} f'(x) = +\infty$ $\lim_{x \to C^+} f'(x) = -\infty$

Example 3: Vertical Tangent or Cusps

Repeat Example 2 for $f(x) = (x+2)^{\frac{2}{3}}$



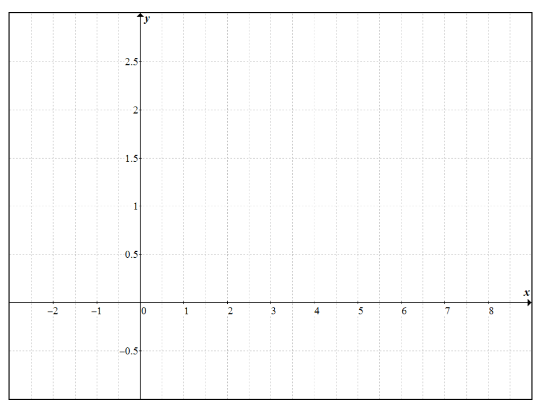
Example 4: Determine the function

Find values for *a*, *b*, and *c* such that the graph of $f(x) = ax^2 + bx + c$ has a relative maximum at (3, 12) and crosses the *y*-axis at (0, 1).

Example 5: Increasing and decreasing function & its local extreme values

Given: $f(x) = x^2 e^{-x}$

a) Determine the domain
b) Find the intervals of increase and decrease.
c) Find the local maximum and local minimum points.
d) Determine the intercepts
e) Sketch the graph



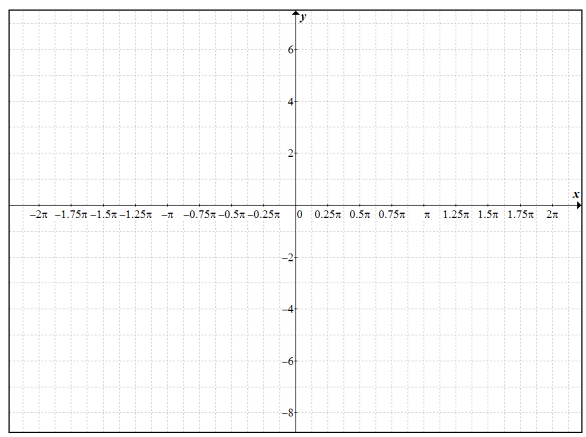
Example 6: Determine the maximum and minimum values of the trig. function

Determine the maximum and minimum values of the function $y = \cos^2 x$ on the interval $x \in [0, 2\pi]$.

Example 7: Increasing and decreasing function & its local extreme values

Given: $f(x) = x - \cos x, -2\pi \le x \le 2\pi$

a) Find the intervals of increase and decrease.
b) Find the local maximum and local minimum points.
c) Sketch the graph



Homework:
P. 193 #2-30, 35-41
Cal & Vectors (Optional)
P. 169 #1,3,5,7,8
P. 178 #5,7,8,9,11,15
P. 233 #12 P. 245 #2,12
P. 256 #6,7,11 P. 260 #5-7