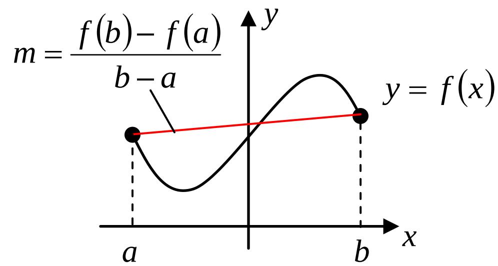
Mean Value Theorem

Date:

Mean Value Theorem (MVT)



Assume that f is **continuous** on the closed interval $[a,b]$ and differentiable on (a,b). Then there exists at least one value c in (a,b) such that: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Note: Consider the **Sandwich Theorem** $x = a \pm 0.001$

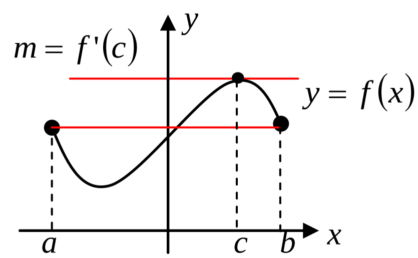
Slope of tangent at $x = a$ can be found by $m = \dfrac{f(a^+) - f(a^-)}{a^+ - a^-}$

Rolle's Theorem

Special case of the MVT where $f(a) = f(b)$. In this case, $f'(c) = 0$.

Suppose that f is continuous on the interval $[a,b]$, differentiable on the interval (a,b) and $f(a) = f(b)$. Then there is a number $c \in (a,b)$ such that $f'(c) = 0$.

Secant Line



Mean Value Theorem (Rolle's Theorem)

Proof

The slope and equation of secant line through ab:

$$m = \frac{f(b) - f(a)}{b - a} \rightarrow m = \frac{f(x) - f(a)}{x - a}$$

$$f(x) = m(x - a) + f(a)$$

Let the function g to be the difference between f and the function whose graph is the secant line, then: $g(x) = f(x) - [m(x - a) + f(a)]$

Note that g is continuous on $[a,b]$ and differentiable on (a,b), since f is.

$$g(a) = f(a) - [m(a - a) + f(a)] \rightarrow 0$$

$$g(b) = f(b) - [m(b - a) + f(a)]$$

$$g(b) = f(b) - [f(b) - f(a) + f(a)] = 0 \because m(b - a) = f(b) - f(a)$$

$\because g(a) = g(b)$, there exists a number c by Rolle's Theorem in the interval (a,b) such that $g'(c) = 0$.

and $g'(x) = f'(x) - m$, $\therefore g'(c) = f'(c) - m$

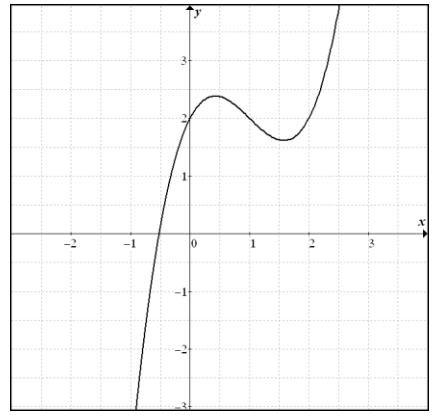
Finally: $f'(c) = m = \dfrac{f(b) - f(a)}{b - a}$ as expected.

Example 1: Mean value Theorem

Illustrate the MVT with $f(x) = \sqrt{x}$ and the points $a = 1$ and $b = 9$.

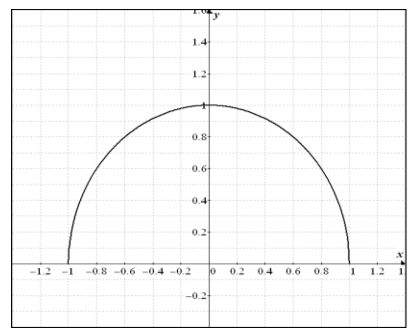
Example 2: An illustration of Rolle's Theorem/MVT

Find a value of c satisfying the conclusion of Rolle's Theorem for $f(x) = x^3 - 3x^2 + 2x + 2$ on interval $[0,1]$.



Example 3: Applying the MVT

Let $f(x) = \sqrt{1 - x^2}$, $A = (-1, f(-1))$, and $B = (1, f(1))$. Find a tangent to f in the interval $(-1,1)$ that is parallel to the secant AB.



Example 4: Finding Every Function with a Given Derivative

Find all functions that have a derivative equal to $3x^2 + 1$

Antiderivative
A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation**.

Example 5: Antiderivative
Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0,2)$.

Example 6: Finding Velocity and Position
Find the velocity and position functions of a body falling freely from a height of 0 meters under each of the following sets of conditions:

a) The acceleration is 9.8 m/sec^2 and the body falls from rest.
b) The acceleration is 9.8 m/sec^2 and the body is propelled downward with an initial velocity of 1 m/sec.

Homework:
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