Power Series

Date:

Infinite Series

A series is the sum of the terms of a sequence. Finite sequences and series have defined first and last terms, whereas infinite sequences and series continue indefinitely.

An infinite series is an expression of the form: $a_1 + a_2 + a_3 + ... + a_n + ...$, or $\sum_{k=1}^{\infty} a_k$.

The numbers $a_1, a_2, a_{3,} ...$ are the terms of the series; a_n is the n^{th} term.

Partial Sum (Infinite Series)

The partial sums of the series form a sequence of real numbers, each defined as a finite sum. If the sequence of partial sums has a limit *S* as $n \to \infty$, we say the series **converges** to the sum *S*, and we write $a_1 + a_2 + a_3 + ... + a_n + ... = \sum_{k=1}^{\infty} a_k = S$.

$$s_1 = a_1$$
$$s_2 = a_1 + a_2 \qquad \Rightarrow s_n = \sum_{k=1}^{n} a_k$$
$$s_3 = a_1 + a_2 + a_3$$
$$\ldots\ldots$$

If the infinite series is converges, then $\lim_{k \to \infty} a_k = 0$. (The very last term is approaching to zero if series is converges ➔ -1 < *r* <1.

Otherwise, we say the series **diverges**.

The numbers $a_1, a_2, ...$ are the terms of the series; a_n is the *n*th term.

Geometric Series

$a + ar + ar^2 + ar^3 + ... + ar^{n-1} + ... = \sum_{n=1}^{\infty} ar^{n-1} (a \neq 0)$ is convergent and has the sum $S = \frac{a}{1-r}$ if $|r| < 1$ and diverges if $|r| \geq 1$.

Example 1: Identify a Divergent Series

Determine whether the following series are convergent or divergent. If it converges, give its sum.

a) $1+1+1+1+...+1+...$

b) $1-1+1-1+...+(-1)^{n+1}+...$

c) $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+...+\frac{3}{10^n}+...$

d) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

e) $\frac{\pi}{2}+\frac{\pi^2}{4}+\frac{\pi^3}{8}+...$

Power Series

An expression of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n + \ldots$ is a **power series** centered at ***x* = 0.**

An expression of the form $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots + c_n(x-a)^n + \ldots$ is a **power series** centered at ***x* = *a*.** The term $c_n(x-a)^n$ is the *n*th term; the number *a* is the center.

Example 2: Finding a Power Series by Differentiation

Given that $\frac{1}{1-x}$ is represented by the power series $1 + x + x^2 + \ldots + x^n + \ldots$ on the interval $(-1,1)$, find a power series to represent a) $\frac{1}{1+x}$ b) $\frac{x}{1+x}$ c) $\frac{1}{1-2x}$ d) $\frac{1}{3x}$ e) $\frac{1}{(1-x)^2}$.

Power Series Date:

Example 3: Finding a Power Series by Integration

Given that $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-x)^n + \ldots$ on the interval $(-1,1)$, find a power series to represent

a) $\ln(1+x)$. b) $\tan^{-1} x$

Radius of Convergence

Let $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$. There are three possibilities:

i) $F(x)$ converges only for $x = c$, or

ii) $F(x)$ converges for all x, or

iii) There is a number $R > 0$ such that $F(x)$ converges absolutely if $|x-c| < R$ and diverges if $|x-c| > R$. It may or may not converge at the endpoints $|x-c| = R$.

In Case (i), set $R = 0$, and in Case (ii), set $R = \infty$. We call R the radius of convergence of $F(x)$.

Homework:
P. 481 #1-37

Taylor Series

Date:

Taylor Series

A Taylor polynomial approximates the value of a function $f(x)$ at the point *x* = *a*. It is a series expansion of a function at a point. If the function and all its derivatives exist at *x* = *a*, then on the interval of convergence, the Taylor series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ converges to $f(x)$.

The **MacLaurin series** is the name given to a Taylor series centered at *x* = 0. $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$

Example 1: Constructing the Taylor Polynomial

Find the Taylor polynomial of degree 3 for $f(x) = \frac{1}{x+2}$ about the point *x* = 3.

Example 2: Constructing a Power series for *sin x*

Construct the seventh Taylor polynomial and the Taylor series for $\sin x$ at *x* = 0. (Maclaurin series)

Recall: $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

Example 3: Applying Taylor Series concepts

A function $f(x)$ is approximated by the **third order** Taylor series $1+2(x-1)-(x-1)^2+(x-1)^3$ centered at $x = 1$. Find $f'(1)$, $f''(1)$ and $f'''(1)$.

Maclaurin Series

The MacLaurin series is the name given to a Taylor series centered at $x = 0$.

$$f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\ldots+\frac{f^{(n)}(0)}{n!}x^n+\ldots=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$$

The partial sum $P_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}x^k$ is the **Taylor polynomial of order *n* for *f* at $x = 0$**

Example 4: Approximating a Function Near 0

Find the fifth order Taylor polynomial that approximates $y=\sin 2x$ near $x = 0$.

Example 5: A Taylor Series at $x = 2$

Find the Taylor series generates by $f(x)=e^x$ at i) $x = 2$; ii) $x=0$ (Maclaurin series)

Example 6: MacLaurin Series

Find the MacLaurin series for the function $f(x) = xe^x$

Maclaurin Series (Taylor Series at $x = 0$ of some common functions)

- $\dfrac{1}{1-x} = 1 + x + x^2 + ... + x^n + ... = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$
- $\dfrac{1}{1+x} = 1 - x + x^2 - ... + (-x)^n + ... = \sum_{n=0}^{\infty} (-1)^n x^n \quad (|x| < 1)$
- $e^x = 1 + x + \dfrac{x^2}{2!} + ... + \dfrac{x^n}{n!} + ... = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ (all real x)
- $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - ... + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + ... = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!}$ (all real x)
- $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - ... + (-1)^n \dfrac{x^{2n}}{(2n)!} + ... = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!}$ (all real x)
- $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - ... + (-1)^{n-1} \dfrac{x^n}{n} + ... = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} \quad (-1 < x \le 1)$
- $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - ... + (-1)^n \dfrac{x^{2n+1}}{2n+1} + ... = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} \quad (|x| \le 1)$

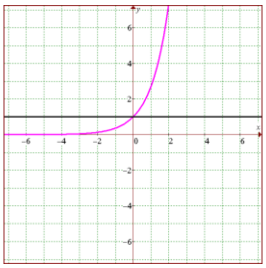
Homework:
P.492 #1-28

Taylor Series

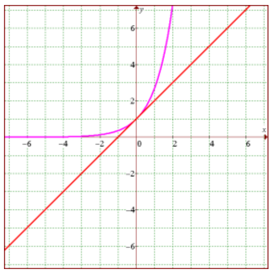
Date:

Taylor Series for $y = e^x$ at $x = 0$ (Maclaurin Series) (Graphical simulations)

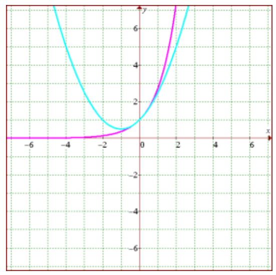
Recall: $e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (all real x)



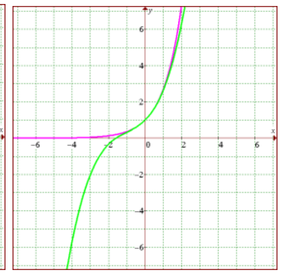
$e^x = 1$
at $x = 0$
when $n = 0$



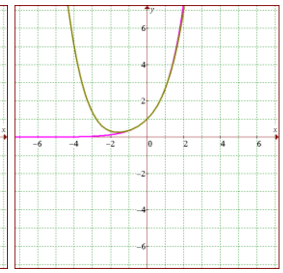
$e^x = 1 + x$
at $x = 0$
when $n = 1$



$e^x = 1 + x + \frac{1}{2}x^2$
at $x = 0$
when $n = 2$



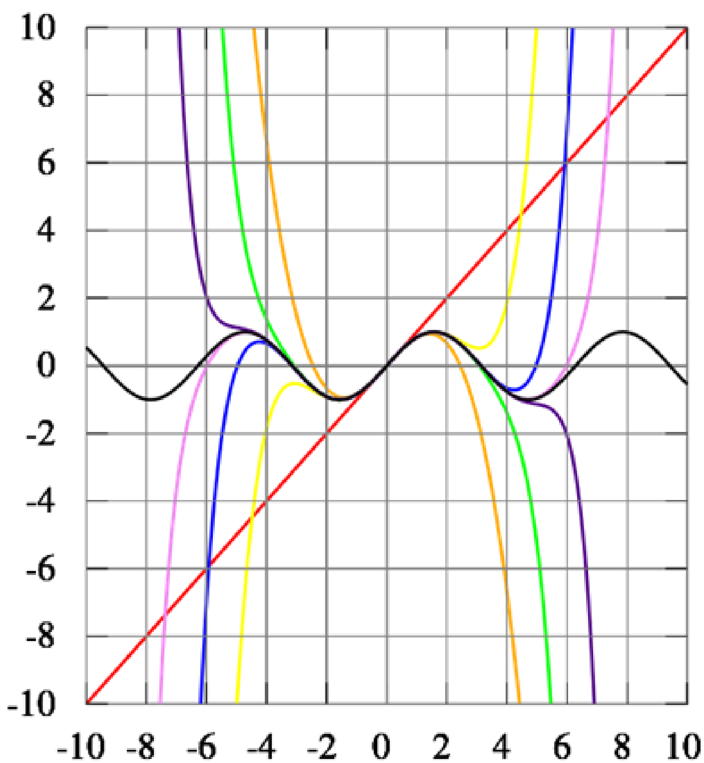
$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$
at $x = 0$
when $n = 3$



$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$
at $x = 0$
when $n = 4$

Taylor Series for $y = \sin x$ at $x = 0$ (Maclaurin Series) (Graphical simulations)

Recall: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ (all real x)



As the degree of the Taylor polynomial rises, it approaches the correct function. This image shows sin(x) and its Taylor approximations, polynomials of degree

***At x* = 0** and

- ➔ $n = 1$: $\sin x = x$
- ➔ $n = 3$: $\sin x = x - \frac{x^3}{3!}$
- ➔ $n = 5$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$
- ➔ $n = 7$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$
- ➔ $n = 9$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$
- ➔ $n = 11$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$
- ➔ $n = 13$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!}$

Taylor's Theorem

Date:

Recall:

Maclaurin Series

The MacLaurin series is the name given to a Taylor series centered at $x = 0$.

$$f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\ldots+\frac{f^{(n)}(0)}{n!}x^n+\ldots=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$$

The partial sum $P_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}x^k$ is the **Taylor polynomial of order *n* for *f* at *x* = 0**

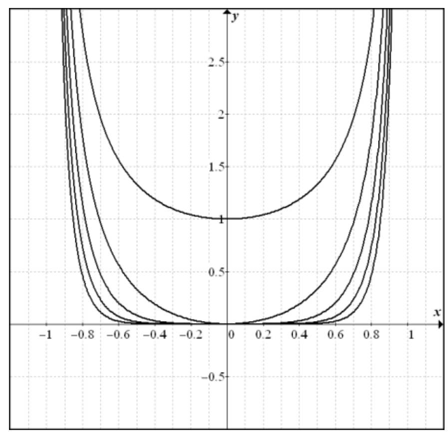
Truncation Error

If we would like to be able to use Taylor polynomials to approximate functions over the intervals of convergence of the Taylor series, and we would like to keep the error of the approximation within specified bounds. Since the error results from truncating the series down to a polynomial, we call it **truncation error**.

Example 1: Truncation Error for a Geometric Series

Find a formula for the truncation error if we use $1+x^2+x^4+x^6$ to approximate $\frac{1}{1-x^2}$ over the interval $(-1,1)$.

Recall: **Maclaurin Series**: $\frac{1}{1-x}=1+x+x^2+\ldots+x^n+\ldots=\sum_{n=0}^{\infty}x^n \quad (|x|<1)$



Taylor's Theorem with Remainder

Every truncation splits a Taylor series into two equally significant pieces: the Taylor polynomial $P_n(x)$ that gives us the approximation, and the remainder $R_n(x)$ that tells us whether the approximation is any good. Taylor's theorem is about both pieces.

If f has derivatives of all orders in an open interval I containing a, then for each positive integer n and for each x in I.

Taylor formula : $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$

$R_n(x)$: Remainder of order n (Error term). It is also called the Lagrange form of the remainder.

where $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ for some c between a and x.

If $R_n(x) \to 0$ as $n \to \infty$ for all x in I, we say that the Taylor series generated by f at $x = a$ converges to f on I, and we write $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

Example 2: Proving Convergence of a Maclaurin Series

Prove that the series $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ converges to $\sin x$ for all real x.

Recall : Maclaurin Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad \text{(all real } x\text{)}$$

Remainder Estimation Theorem

If there are positive constants M and r such that $\left|f^{(n+1)}(t)\right| \le Mr^{n+1}$ for all t between a and x, then the remainder $R_n(x)$ in Taylor's Theorem satisfies the inequality $\left|R_n(x)\right| \le M\,\frac{r^{n+1}\left|x-a\right|^{n+1}}{(n+1)!}$.

If these conditions hold for every n and all the other conditions of Taylor's Theorem are satisfied by f, then the series converges to $f(x)$

Example 3: Determine the accuracy of the approximation

Use Taylor's Theorem to approximate sin(0.1) by $P_3(0.1)$ and determine the accuracy of the approximation.

Recall **Maclaurin Series**:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \quad \text{(all real } x\text{)}$$

Example 4: Proving Convergence of a Maclaurin Series

Prove that the series $f(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}$ converges to e^x for all real *x*.

Recall **Maclaurin series**:

$$e^x=1+x+\frac{x^2}{2!}+...+\frac{x^n}{n!}+...=\sum_{n=0}^{\infty}\frac{x^n}{n!} \quad \text{(all real } x\text{)}$$

Example 5: Euler's Formula

Try Exploration 2 Text book: P. 499

a) Find the Maclaurin series for e^{ix}

b) Use the result in part *a* to prove $e^{ix}=\cos x+i\sin x$ which is known as **Euler's formula**.

c) Use the Euler's formula to prove that $e^{i\pi}+1=0$

Homework
P. 500 #1 - 36

Radius of Convergence /Testing Convergence at Endpoints Date:

Convergence

A convergent series is a number and may be treated as such
A divergent series is not a number and must not be treated as one.

- Equality
  $1 + 1 = 2$ signifies *equally of real numbers.* It is a true sentence.

- Equivalent Expression
  $2(x-3) = 2x-6$ signifies *equivalent expressions.* It is a true sentence.

- Equation
  $x^2 + 3 = 7$ is an *equation*. It is an *open sentence*, because it can be true or false depending on the value of *x*.

- Identity
  $\frac{(x^2-1)}{x+1} = x-1$ is an *identity*. It is a true sentence, but *x* must be in the domain of both expression. $x \neq -1$

Example 1: The importance of Convergence

Consider the sentence $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots + (-1)^n x^{2n} + \ldots$

For what value of *x* is this an identity?

$$\frac{1}{1+x} = 1 - x + x^2 - \ldots + (-x)^n + \ldots = \sum_{n=0}^{\infty} (-1)^n x^n \quad (|x| < 1) \text{ or } -1 < x < 1$$

Convergence Theorem for Power Series

There are three possibilities for $\sum_{n=0}^{\infty} c_n (x-a)^n$ with respect to convergence:

1. There is a positive number *R* such that the series diverges for $|x-a| > R$ but converges for $|x-a| < R$. The series may or may not converge at either of the endpoints $x = a - R$ and $x = a + R$.
2. The series converges for every *x* $(R = \infty)$
3. The series converges at $x = a$ and diverges elsewhere ($R = 0$)

The number *R* is the **radius of convergence**, and the set of all values of *x* for which the series converges is the **interval of convergence**.

n^{th} – Term Test

If $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(S_n - S_{n-1}\right) = S - S = 0$, then the series converge.

Direct Comparison Test

Let $\sum a_n$ be a series with no negative terms.

$\sum a_n$ converges if there is a convergent series $\sum c_n$ with $a_n \le c_n$ for all $n > N$, for some integer N.

Proof:

For $k \le N$, it is obvious that $S_k = a_1 + a_2 + ... + a_k \le a_1 + a_2 + ... + a_N + \sum_{n=N+1}^{\infty} c_n$

For $k > N$, $S_k = a_1 + a_2 + ... + a_k = a_1 + ... + a_N + a_{N+1} + ... + a_k$

$$\le a_1 + ... + a_N + c_{N+1} + ... + c_k$$

$$\le a_1 + ... + a_N + \sum_{n=N+1}^{\infty} c_n$$

Since all of the a_n are nonnegative, the partial sums of the series form a nondecreasing sequence of real numbers. Therefore it shows that this sequence is bounded above, so it must converge to limit.

$\sum a_n$ diverges if there is a convergent series $\sum d_n$ of nonnegative terms with $a_n \ge d_n$ for all $n > N$, for some integer N.

Proof:

For $k \le N$, it is obvious that $S_k = d_1 + d_2 + ... + d_k \le d_1 + d_2 + ... + d_N + \sum_{n=N+1}^{\infty} a_n$

For $k > N$, $S_k = d_1 + d_2 + ... + d_k = d_1 + ... + d_N + d_{N+1} + ... + d_k$

$$\le d_1 + ... + d_N + a_{N+1} + ... + a_k$$

$$\le d_1 + ... + d_N + \sum_{n=N+1}^{\infty} a_n$$

If $\sum a_n$ converged, it would imply that $\sum d_n$ was also convergent.

Example 2: Proving Convergence by Comparison

Prove that $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$ converges for all real x.

Recall: $e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ... = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (all real x)

Recall:

$e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ... = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

(all real x)

Absolute Convergence Implies Convergence

If the series $\sum |a_n|$ of absolute values converges, then $\sum a_n$ **converges absolutely**.

If $\sum |a_n|$ converges, then $\sum a_n$ converges

Example 3: Using Absolute Convergence

Show that $\displaystyle\sum_{n=0}^{\infty} \frac{(\sin x)^n}{n!}$ converges for all x.

Recall:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

(all real x)

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

Ratio Test (See proofs in text book P. 507)

Let $\sum a_n$ be a series with positive terms, and with $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

Then,

a) The series converges if $L < 1$,
b) The series diverges if $L > 1$,
c) the test is inconclusive if $L = 1$.

Example 4: Finding the Radius of Convergence

Find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$

Example 5: Finding the Radius of Convergence

Find the radius of convergence of $\sum_{n=0}^{\infty} n!x^n$

Example 6: Determining Convergence of a Series

Determine the convergence or divergence of the series $\sum_{n=0}^{\infty} \frac{3^n}{5^n+1}$

Homework:
P. 511 # 7 - 43

The Integral Test
Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where *f* is a continuous, positive, decreasing function of *x* for all $x \geq N$ (*N* a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)dx$ either both converge or both diverge.

Example 7: Applying the Integral Test

Does $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ converge?

The *p*-series Test (Harmonic Series)

$\sum_{n=1}^{\infty}\left(\frac{1}{n^p}\right)$ converges if $p > 1$, diverges if $P < 1$ or $P = 1$.

Proof:

$f(x)=\frac{1}{x^p}$ $\int_1^{\infty}\frac{1}{x^p}dx$

Case 1: *p* > 1

$$\int_1^{\infty}x^{-p}dx=\left.\frac{x^{1-p}}{-p}\right]_1^{\infty}=\frac{\infty^{1-p}}{-p}-\frac{1}{-p}$$

$$=0+\frac{1}{p} \quad \therefore (Converge)$$

Case 2: *p* < 1

$$\int_1^{\infty}x^{-p}dx=\left.\frac{x^{1-p}}{-p}\right]_1^{\infty}=\frac{\infty}{-p}-\frac{1}{-p}$$

$$=\infty \quad \therefore (Diverge)$$

Case 3: *p* = 1

$$\int_1^{\infty}\frac{1}{x}dx=\ln x\Big]_1^{\infty}=\ln\infty-\ln 1$$

$$=\infty-0$$

$$=\infty \quad \therefore (Diverge)$$

The Limit Comparison Test (LCT)

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \ge N$ (*N* a positive integer)

If $\lim_{n\to\infty}\frac{a_n}{b_n}=c,\ 0<c<\infty$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

If $\lim_{n\to\infty}\frac{a_n}{b_n}=0$, and $\sum b_n$ converges, then $\sum a_n$ converges.

.If $\lim_{n\to\infty}\frac{a_n}{b_n}=\infty$, and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Example 8: Using the Limit Comparison Test

Determine whether the series converge or diverge.

a) $\frac{3}{4}+\frac{5}{9}+\frac{7}{16}+\frac{9}{25}+...=\sum_{n=1}^{\infty}\frac{2n+1}{(n+1)^2}$

b) $\frac{1}{1}+\frac{1}{3}+\frac{1}{7}+\frac{1}{15}+...=\sum_{n=1}^{\infty}\frac{1}{2^n-1}$

c) $\frac{8}{4}+\frac{11}{21}+\frac{14}{56}+\frac{17}{115}+\ldots=\sum_{n=2}^{\infty}\frac{3n+2}{n^3-2n}$

d) $\sin 1+\sin\frac{1}{2}+\sin\frac{1}{3}+\ldots=\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$

Alternating Series Test (Leibniz's Theorem)

The series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k = a_1 - a_2 + a_3 - a_4 + \ldots$ converges if the following conditions are satisfied:

1) $0 < a_{k+1} \le a_k$ for all $k \ge \text{integers } N$.
2) $\lim_{k\to\infty} a_k = 0$

Example 9: Alternating Series Test

Investigate the convergence or divergence of the alternating series with $(-1)^k$ in the series.

a) $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}$ b) $\sum_{k=3}^{\infty}\frac{(-1)^k k}{k+2}$ c) $\sum_{k=1}^{\infty}\frac{(-1)^k(k+3)}{k(k+1)}$

Example 10: Finding Intervals of Convergence

For what values of *x* do the following series converge?

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{2n}=\frac{x^2}{2}-\frac{x^4}{4}+\frac{x^6}{6}-\ldots$

b) $\sum_{n=0}^{\infty}\frac{(10x)^n}{n!}=1+10x+\frac{100x^2}{2!}+\frac{1000x^3}{3!}+\ldots$

c) $\sum_{n=0}^{\infty}n!(x+1)^n=1+(x+1)+2!(x+1)^2+3!(x+1)^3+\ldots$

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