Exponential & Logarithmic Reference Sheet

Date:

Exponent Laws

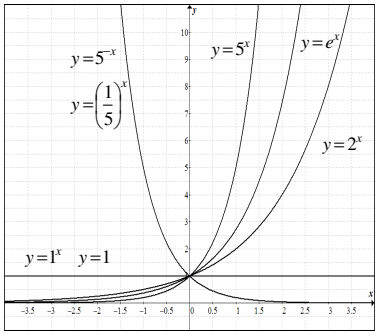
- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $a^0 = 1$

Natural Exponential Number

$$e = \lim_{x\to 0}(1+x)^{\frac{1}{x}} \text{ or } \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

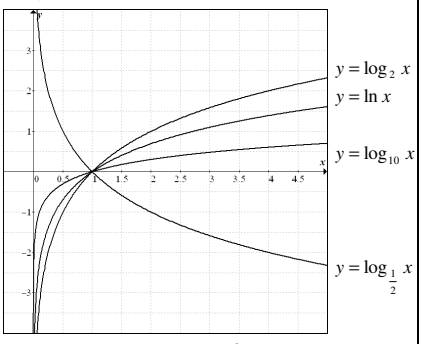
$\approx 2.718281828459.....$

Exponential Functions



- Domain is $x \in R$
- Range is $y \in R \mid y > 0$
- y-intercept at 1
- $e^{\ln x} = x, x > 0$
- Horizontal Asymptote: $y = 0$.

Logarithmic Functions



- Domain is $x \in R \mid x > 0$
- Range is $y \in R$
- x-intercept at 1
- $\ln e^x = x, x \in R$
- Vertical Asymptote: $x = 0$.

Basic Properties

1) $\log_e 1 = 0 \quad \Leftrightarrow \ln 1 = 0$

2) $\log_e e = 1 \quad \Leftrightarrow \ln e = 1$

3) $\log_e e^x = x \quad \Leftrightarrow \ln e^x = x$

4) $e^{\log_e x} = x \quad \Leftrightarrow e^{\ln x} = x \quad (x > 0)$

5) $\log x = \log_{10} x$

Exponential ⟷ Logarithmic

$y = \log_b x \Leftrightarrow b^y = x$

$y = \ln x \Leftrightarrow e^y = x$

Change of base

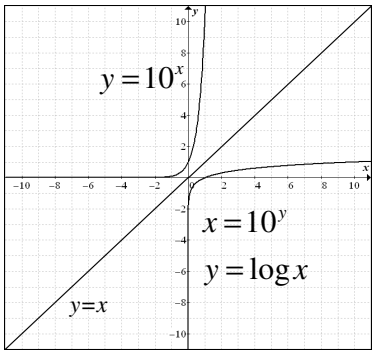
1) $\log_a b \Leftrightarrow \frac{\log b}{\log a} \Leftrightarrow \frac{\log_e b}{\log_e a} \Leftrightarrow \frac{\ln b}{\ln a}$

2) $\log_a b = \frac{1}{\log_b a}$

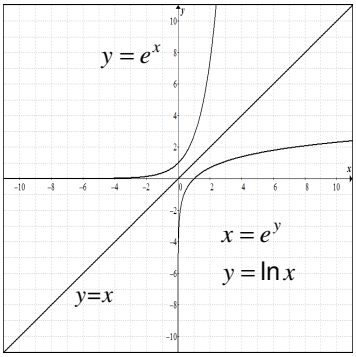
Laws of Logarithm

Product Law:	$\log_b xy = \log_b x + \log_b y$	$\ln xy = \ln x + \ln y$
Quotient Law:	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\ln \frac{x}{y} = \ln x - \ln y$
Power Law:	$\log_b x^p = p \log_b x$	$\ln x^p = p \ln x$

Exp & Log Functions



Natural (Exp & Log) Functions



Exponential Growth

$y_t = y_o(1+r)^t \Leftrightarrow y_t = y_o R^t, R > 1, R = (1+r)$

Exponential Decay

$y_t = y_o(1-r)^t \Leftrightarrow y_t = y_o R^t, 0 < R < 1, R = (1-r)$

y_0 = original value r = growth % y_t = value after time t

Derivatives of Exponential

$$\frac{d}{dx}e^{g(x)} = e^{g(x)}g'(x)$$

$$\frac{d}{dx}b^{g(x)} = b^{g(x)}g'(x)\ln b$$

Derivatives of Logarithm

$$\frac{d}{dx}\ln g(x) = \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}\log_b g(x) = \frac{g'(x)}{g(x)\ln b}$$

Doubling Period (d)

$y_t = y_o(2)^{\frac{t}{d}}$

Half-life (*h*)

$y_t = y_o(0.5)^{\frac{t}{h}}$

Compound Interest

$$A = P(1+i)^k = P\left(1+\frac{r}{n}\right)^{nt}$$

P: Principal
A: Amount after t yrs
k: Periods in t years
r: Interest rate per annum
n: Periods in 1 year

Limits of Exponential

$\lim_{x\to-\infty} b^x = \infty, 0 < b < 1$

$\lim_{x\to\infty} b^x = -\infty, 0 < b < 1$

$\lim_{x\to-\infty} b^x = 0, b > 1$

$\lim_{x\to\infty} b^x = \infty, b > 1$

Limits of Logarithmic

$\lim_{x\to 0^+} \ln x = -\infty$

$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to 0^+} \log_b x = -\infty, b > 1$

$\lim_{x\to\infty} \log_b x = \infty, b > 1$