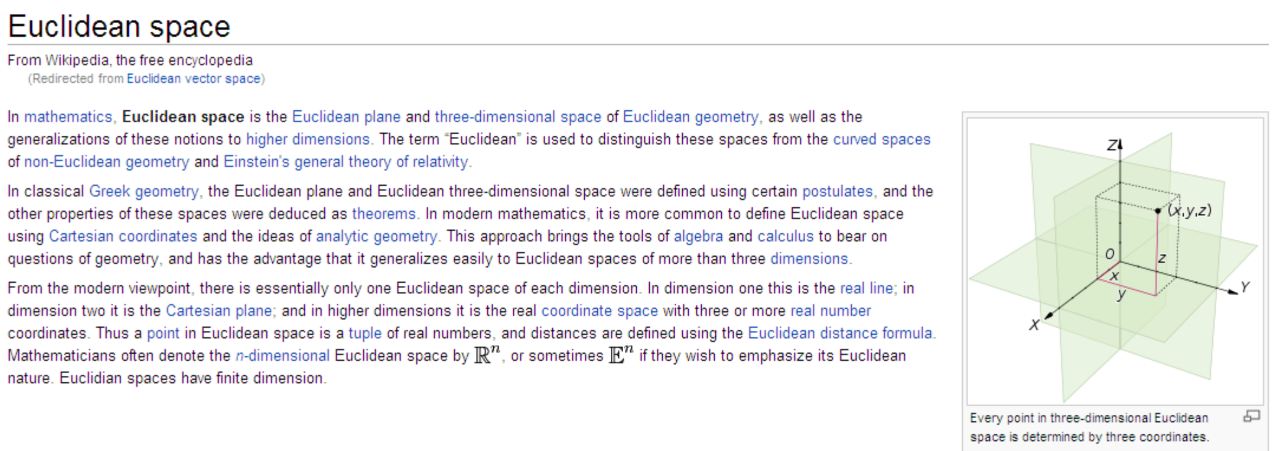
Dot Product

The idea of dot product is to find the angle between two vectors, the principal use of this product is the **inner product** in a Euclidean vector space: when two vectors are expressed on an orthonormal basis, the dot product of their coordinate vectors gives their inner product.



Euclidean space

From Wikipedia, the free encyclopedia
(Redirected from Euclidean vector space)

In mathematics, **Euclidean space** is the Euclidean plane and three-dimensional space of Euclidean geometry, as well as the generalizations of these notions to higher dimensions. The term "Euclidean" is used to distinguish these spaces from the curved spaces of non-Euclidean geometry and Einstein's general theory of relativity.

In classical Greek geometry, the Euclidean plane and Euclidean three-dimensional space were defined using certain postulates, and the other properties of these spaces were deduced as theorems. In modern mathematics, it is more common to define Euclidean space using Cartesian coordinates and the ideas of analytic geometry. This approach brings the tools of algebra and calculus to bear on questions of geometry, and has the advantage that it generalizes easily to Euclidean spaces of more than three dimensions.

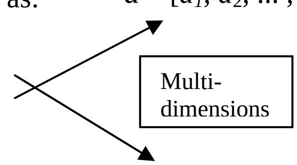
From the modern viewpoint, there is essentially only one Euclidean space of each dimension. In dimension one this is the real line; in dimension two it is the Cartesian plane; and in higher dimensions it is the real coordinate space with three or more real number coordinates. Thus a point in Euclidean space is a tuple of real numbers, and distances are defined using the Euclidean distance formula. Mathematicians often denote the n-dimensional Euclidean space by $\mathbb{R}^n$, or sometimes $\mathbb{E}^n$ if they wish to emphasize its Euclidean nature. Euclidian spaces have finite dimension.

Every point in three-dimensional Euclidean space is determined by three coordinates.

The dot product of two vectors $\vec{a} = [a_1, a_2, \ldots, a_n]$ and $\mathbf{b} = [b_1, b_2, \ldots, b_n]$ is defined as:

$$\mathbf{a}\cdot\mathbf{b} = \sum_{i=1}^{n} a_i b_i = a_1b_1 + a_2b_2 + \cdots + a_nb_n$$

$\vec{a} = [a_1, a_2, \ldots, a_n]$

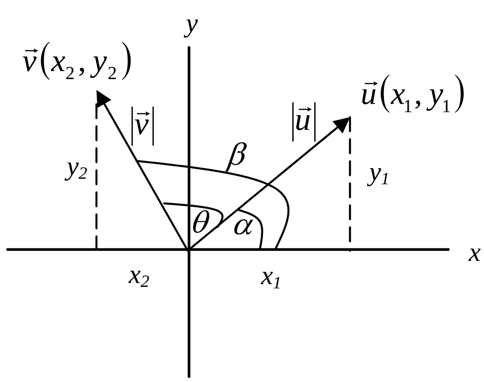


$\vec{b} = [b_1, b_2, \ldots, b_n]$

Proof: $|\vec{u}||\vec{v}|\cos\theta = \vec{u}\cdot\vec{v}$

So in R_2, let's start with proving $|\vec{u}||\vec{v}|\cos\theta = x_1y_1 + x_2y_2 = \vec{u}\cdot\vec{v}$ where $\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$

Recall in 2D, : $\vec{u}\cdot\vec{v} = x_1x_2 + y_1y_2$



$$\begin{aligned}
|\vec{u}||\vec{v}|\cos\theta &= |\vec{u}||\vec{v}|\cos(\beta-\alpha)\\
&= |\vec{u}||\vec{v}|(\cos\beta\cos\alpha + \sin\beta\sin\alpha)\\
&= |\vec{u}||\vec{v}|\left[\left(\frac{x_2}{|\vec{v}|}\right)\left(\frac{x_1}{|\vec{u}|}\right) + \left(\frac{y_2}{|\vec{v}|}\right)\left(\frac{y_1}{|\vec{u}|}\right)\right]\\
&= |\vec{u}||\vec{v}|\left(\frac{x_1x_2 + y_1y_2}{|\vec{u}||\vec{v}|}\right)\\
&= x_1x_2 + y_1y_2\\
&= \vec{u}\cdot\vec{v}
\end{aligned}$$

Dot product in component form (2D)

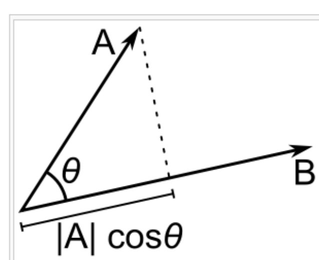
$\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$

$\vec{u}\cdot\vec{v} = (x_1)(x_2) + (y_1)(y_2)$

Dot product in component form (3D)

$\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$

$\vec{u}\cdot\vec{v} = (x_1)(x_2) + (y_1)(y_2) + (z_1)(z_2)$



A • B = |A| |B| cos(θ).
|A| cos(θ) is the scalar projection of A onto B.