A: 37/37 Lines and Planes Name:

1. Determine the vector, parametric and symmetric equations of the line passing through the points $A(-3,2,8)$ and $B(4,3,6)$.

$\vec{d_1} = \vec{AB} = (4,3,6)-(-3,2,8)$
$= (7,1,-2)$

vector equation:
$\vec{r} = (-3,2,8) + t(7,1,-2)$, $t \in \mathbb{R}$

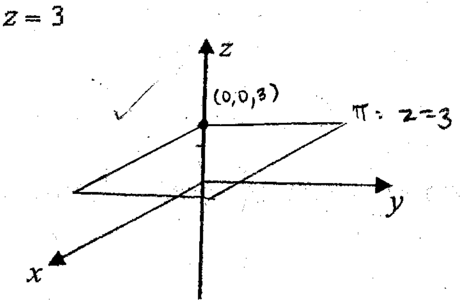
parametric equations:
$x = -3+7t$
$y = 2+t$, $t \in \mathbb{R}$
$z = 8-2t$

symmetric equation:
$t = \dfrac{x+3}{7} = \dfrac{y-2}{1} = \dfrac{z-8}{-2}$, $t \in \mathbb{R}$

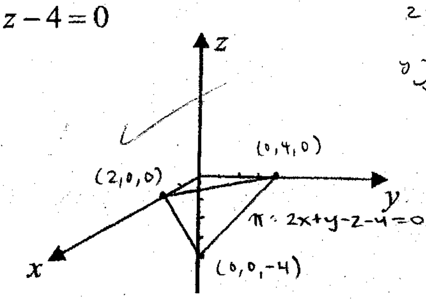
(4)

2. Sketch the following planes. Be sure to label your sketches.

a) $z = 3$



b) $2x+y-z-4=0$



x-int:
$2x = 4$
$x = 2$
y-int:
$y = 4$
z-int:
$-z = 4$
$z = -4$

(2) (2)

3. Determine the Cartesian equation of the plane having x-, y-, and z-intercepts of 2, 5, and 3 respectively

$X(2,0,0)$
$Y(0,5,0)$
$Z(0,0,3)$

$\vec{d_1} = \vec{XY} = (0,5,0)-(2,0,0)$
$= (-2,5,0)$

$\vec{d_2} = \vec{YZ} = (0,0,3)-(0,5,0)$
$= (0,-5,3)$

π: $\vec{r} = (2,0,0) + s(-2,5,0) + t(0,-5,3)$

$\vec{n} = \vec{d_1} \times \vec{d_2}$
$= (-2,5,0)\times(0,-5,3)$
$= \begin{matrix} -2 & 5 & 0 & -2 & 5 \\ 0 & -5 & 3 & 0 & -5 \end{matrix}$
$= (15,6,10)$

$\therefore \pi: 15x+6y+10z+D=0$
sub in $(2,0,0)$:
$15(2)+6(0)+10(0)+D=0$
$30+D=0$
$D=-30$

$\boxed{\therefore \pi: 15x+6y+10z-30=0}$

(5)

4. Find the point of intersection, if any, of the lines $l_1: \dfrac{x+5}{3} = \dfrac{y-2}{2} = \dfrac{z+7}{6}$ and $l_2: x = \dfrac{y+6}{-5} = \dfrac{z+3}{-1}$.

Check your solution.

l_1: $x = -5+3s$
$y = 2+2s$
$z = -7+6s$

l_2: $x = t$
$y = -6-5t$
$z = -3-t$

$-5+3s = t$ ①
$2+2s = -6-5t$ ②
$-7+6s = -3-t$ ③

①→②:
$2+2s = -6-5(-5+3s)$
$2+2s = -6+25-15s$
$17s = 17$
$s = 1$ ④

④→①:
$t = -5+3(1)$
$= -2$ ⑤

check ④, ⑤ in ③:

LS	RS
$=-7+6s$	$=-3-t$
$=-7+6(1)$	$=-3-(-2)$
$=-1$	$=-1$

$\because LS = RS$
$\therefore s=1$, $t=-2$

Determine Point of Intersection:
$x = -5+3(1)$
$= -2$
$y = 2+2(1)$
$= 4$
$z = -7+6(1)$
$= -1$

check point of intersection:
$x = -2$
$y = -6-5(-2)$
$= 4$
$z = -3-(-2)$
$= -1$

$\boxed{\therefore \text{point of intersection is } (-2,4,-1)}$

(5)

5. Determine possible values of k such that the two planes are coincident.

$\pi_1: x+3y-z=9$
$\pi_2: 3x+k^2y-3z=k^3$

to be coincident, must be parallel so $\vec{n_1} = m\vec{n_2}$

$(1,3,-1) = m(3,k^2,-3)$
$1 = 3m$ ①
$3 = k^2m$ ②
$-1 = -3m$ ③
①: $m = \frac{1}{3}$
③: $m = \frac{1}{3}$
①,③→②:
$3 = k^2(\frac{1}{3})$
$9 = k^2$
$k = \pm 3$

to be coincident, ratio between D values must be the same as ratio between coefficients

$9 = k^3m$
$9 = k^3(\frac{1}{3})$
$27 = k^3$
$k = 3$

$\boxed{\therefore \text{for the two planes to be coincident, } k \text{ must equal } 3}$

(3)

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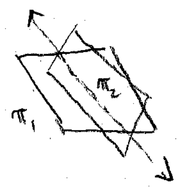
6. Describe the intersections of the following, and justify your answers. (drawings are optional but appreciated) **<u>DO NOT SOLVE</u>**

a) $\pi_1: \; x-3y+2z-5=0$
$\pi_2: \; 2x-y+4z+7=0$

(2) $\pi_1 \not\parallel \pi_2$

$\pi_1 \neq \pi_2$

(2) ∴ two non parallel, non coincident planes intersect in a line

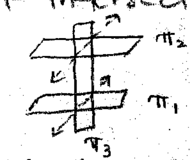


b) $\pi_1: \; x-3y+2z-5=0$
$\pi_2: \; 2x-6y+4z+7=0$
$\pi_3: \; 3x+y-2z+11=0$

$\pi_1 \parallel \pi_2$

$\pi_1 \neq \pi_2 \neq \pi_3$

∴ π_1 and π_2 are two parallel and distinct planes. π_3 intersects both planes creating two different lines. All 3 planes do not intersect.



7. Solve the following system of equations. State the solution and give the geometric interpretation of the system.

$\pi_1: x+2y+3z=-4$
$\pi_2: 2x+3y+4z=-5$
$\pi_3: 3x+4y+5z=-6$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 2 & 3 & 4 & -5 \\ 3 & 4 & 5 & -6 \end{array}\right] \Rightarrow \begin{array}{c} R1 \\ 2R1-R2 \\ 3R1-R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 4 & -6 \end{array}\right] \Rightarrow \begin{array}{c} R1 \\ R2 \\ R3 \div 2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 2 & -3 \end{array}\right]$$

$$\Rightarrow \begin{array}{c} R1 \\ R2 \\ R2-R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & -4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

∴ $0z=0$
∵ always true
let $z=t$

$y+2z=-3$
$y+2t=-3$
$y=-3-2t$

$x+2y+3z=-4$
$x+2(-3-2t)+3t=-4$
$x-6-4t+3t=-4$
$x=2+t$

(5) ∴ the intersection is $\vec{r}=(2,-3,0)+t(1,-2,1)$, $t\in\mathbb{R}$

Three nonparallel, non coincident planes intersect in a line

8. Given three planes:
$\pi_1: \; x-y+z=1$
$\pi_2: \; x+y+2z=2$
$\pi_3: \; x+ky-z=1$

Determine a value of k such that these planes intersect in a triangular prism. (also known as the "Toblerone Effect" ;-)

(4)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & k & -1 & 1 \end{array}\right] \Rightarrow \begin{array}{c} R1 \\ R1-R2 \\ R1-R2 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & -2 & -1 & -1 \\ 0 & -1-k & 2 & 0 \end{array}\right] \Rightarrow \begin{array}{c} R1 \\ R2 \\ 2R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & -2 & -1 & -1 \\ 0 & -5-k & 0 & -2 \end{array}\right]$$

if the planes intersect in a triangular prism then their normals are coplanar

$0=\vec{n_1}\times\vec{n_2}\cdot\vec{n_3}$

$0=(1,-1,1)\times(1,1,2)\cdot(1,k,-1)$

$=\begin{vmatrix} -1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 1 \end{vmatrix}\cdot(1,k,-1)$

$=(-3,-1,2)\cdot(1,k,-1)$

$=-3-k-2$

$5=-k$

$k=-5$

∴ for the planes to intersect in a triangular prism, k must be -5

9. Determine the intersection of the line $x=4+2t$, $y=1-2t$, $z=2+3t$ and the plane $2x+5y-z=29$

Determine the value of t:

substitute the line into the plane

(3) $2(4+2t)+5(1-2t)-(2+3t)=29$

$8+4t+5-10t-2-3t=29$

$-9t=18$

$t=-2$ ①

sub ① into the parametric equations for the point of intersection:

$x=4+2(-2)$
$=0$

$y=1-2(-2)$
$=5$

$z=2+3(-2)$
$=-4$

∴ the point of intersection is $(0,5,-4)$

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